Limited-Feedback Transmit Diversity Schemes: Proposed Methods And Impact Of Feedback Delay And Channel Estimation Errors

by

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A thesis submitted to the Department of
Electrical and Computer Engineering
in conformity with the requirements for
the degree of Master of Science (Engineering)

Queen's University
Kingston, Ontario, Canada
January 2007

Your file Votre référence
ISBN: 978-0-494-26547-5
Our file Notre référence
ISBN: 978-0-494-26547-5

Abstract

In this thesis we propose a practical hybrid transmit diversity scheme that applies phase adjustment on signals of stronger subchannels using a few bits of channel information. This scheme requires considerably reduced electronic circuitry at the transmitter compared to the partial phase combining (PPC) and equal gain transmission (EGT) schemes and can outperform them by avoiding energy waste over insignificant subchannels. In 2-transmit antenna systems, the proposed hybrid method adjusts the power of transmit signals using one bit magnitude information along with applying PPC. This scheme offers a very close performance to the optimal beamforming scheme (OBS) by using very few feedback bits and maintaining a simple implementation. We also propose an FFT-based transmit antenna selection scheme which substantially outperforms the conventional antenna selection scheme in correlated fading channels and systems with uniform linear array of omnidirectional antennas. We propose a fast almost-optimal procedure for codeword extraction of PPC method which significantly reduces computational complexity.

We analyze the joint and separate impact of channel estimation errors and feedback delay on the performance of PPC, hybrid and transmit antenna selection schemes. We derive the average SNR expression for the these methods under channel estimation errors and feedback delay. We show that the channel estimation errors and feedback

delay have separable impacts on the average SNR performance and the average SNR degradation due to estimation errors is identical in these methods. We also derive the exact closed-form expressions for the average BER of transmit antenna selection under channel estimation errors and feedback delay in the 2×1 case. We show that for all the transmit diversity schemes, channel estimation errors do not change the diversity gain and degrade the performance by reducing the coding gain. We also show that feedback delay results in diversity gain reduction and if the channel estimation is perfect, feedback delay reduces the diversity gain to 1.

Acknowledgments

First and foremost, I would like to express my sincerest gratitude to my supervisor Professor Saeed Gazor for his invaluable support and insightful guidance throughout my master's studies at Queen's university. I am always thankful for his kindness, motivation and all I have learned from him during these past two years. This would not be possible without him.

I wish to thank the other members of my thesis committee: Professor Shahram Yousefi, Professor Navin Kashyap, Professor Thomas R. Dean and Professor Scott Parent for having accepted to take the time out of their busy schedules to read my thesis and provide me with their invaluable comments. I would also like to thank the ECE staff specially Bernice Ison and Debie Fraser and my colleagues in the Advanced Multidimensional Signal Processing Lab for their kindness and friendship.

I would not have been here today if it were not for the love and support of my family. I want to express my deepest thanks to my father, Morteza, who has always been my greatest mentor and supporter and to my mother, Shokoufeh, whose unconditional love, sacrifice and care are beyond words.

And my most special thanks to my husband, Saeed, for his generous love, support and patience, for helping and encouraging me through the difficulties of my academic experience and above all, for always being by my side through this journey of my life.

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List of Abbreviations

Abbreviation	Description
BER	Bit Error Rate
BPSK	Binary Phase-Shift Keying
cdf	Cumulative Distribution Function
CSCG	Circularly Symmetric Complex Gaussian
CSI	Channel State Information
EGT	Equal Gain Transmission
FFT	Fast Fourier Transform
LOS	Line Of Sight
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
MRC	Maximal Ratio Combining
MRT	Maximal Ratio Transmission
OBS	Optimal Beamforming Scheme
OSTBC	Orthogonal Space Time Block Code
OT	Orthogonal Transformation
pdf	Probability Distribution Function
PPC	Partial Phase Combining
QoS	Quality of Service
QPSK	Quadrature Phase-Shift Keying
RF	Radio-Frequency
SIMO	Single-Input Multiple-Output
SISO	Single-Input Single-Output
SNR	Signal-to-Noise-Ratio
SVQ	Sphere Vector Quantization
TNR	Transmit-power-to-Noise-Ratio
UMTS	Universal Mobile Telecommunications System
VQ	Vector Quantization
WCDMA	Wide-band Code Division Multiple Access

Chapter 1

Introduction

1.1 Introduction and Motivation

Wireless systems continue to strive for ever higher data rates. This goal is particularly challenging for systems that are power, bandwidth, and complexity limited. However, another domain can be exploited to significantly increase channel capacity: the use of multiple transmit and receive antennas. MIMO (multiple-input multiple-output) technology provides significant increase in systems capacity and bandwidth efficiency as well as in Quality of Service (QoS) and reliability in wireless communications through the use of spatial-temporal parallel processing at both the transmitter and receiver [1], [2]. These technologies exploit the spatial diversity in a multiple-antenna wireless system to combat the multipath fading effects of the wireless channel. These benefits are a direct result of the fact that sufficiently spaced antennas encounter approximately independent fading channels.

Receive diversity can be applied at the base station receivers to increase the capacity of uplink (mobile to base-station) but it is practically difficult to be applied in downlink (base-station to mobile) applications due to the implementation complexity increase and size limitation of the mobile terminal. Most of the current handheld devices can accommodate only one or at most two antennas because of the size and power limitations. Hence for downlink applications, multiple antennas can be used at the transmitter instead of the receiver to obtain spatial diversity which is called transmit diversity. Transmit antenna diversity as a means of improving the base station to mobile performance has been an active area of research lately and is in particular attractive because the antennas can be located at the base station and the benefits shared by all users. These are our motivations to focus on transmit diversity schemes for multiple-input single-output (MISO) wireless systems and two-transmit single-receive (2×1) antenna systems as a practical example.

Transmit diversity schemes can be broadly divided into two categories, the ones requiring mobile to base station feedback, called the closed-loop transmit diversity schemes, and the ones not requiring the mobile to base station feedback, referred to as the open-loop transmit diversity schemes. In the closed-loop transmit diversity schemes, the channel state information (CSI) is provided at the transmitter through the feedback channel, and in the open-loop schemes the transmitter has no knowledge of the CSI. The closed-loop schemes utilize the CSI to adjust the transmitted signals to the channel conditions such that the received signal to noise ratio (SNR) and the capacity increase, and this enhanced performance depends on the amount of channel information available at the transmitter. In addition to the two extreme cases of perfect CSI and no CSI at the transmitter, in many realistic situations the transmitter may have only partial CSI, for example, due to a limited capacity of feedback channel or rapid channel variations. In these cases, extra throughput is gained by adapting the transmitter to the typical channel. The idea of exploiting partial (or statistical) information of the channel at the transmitter was proposed by Visotsky *et al.* [3], Jafar *et al.* [4] and Narula *et al.* [5] for the MISO case.

In most transmit diversity schemes with partial CSI, some form of CSI is fed back to the transmitter as an index in a pre-determined codebook. The codebook is known at both the receiver and transmitter and is designed to satisfy the performance criteria. Due to the limitation of feedback channel capacity, it is important to use the form of CSI that is most efficient and captures the essential information of the channel that is critical for channel capacity or error rate performance. For this reason, in this thesis we examine the deployment of the phase information and the magnitude information about the channel coefficients separately and jointly in order to determine the value and importance of each of these two components of CSI. We examine the antenna selection scheme [6], which only uses the magnitude information, the partial phase combining (PPC) scheme [7], which only applies the phase information of the channel coefficients as two extreme cases and propose a hybrid method which deploys both the magnitude and the phase information of the channel coefficients. In order to evaluate the effect of each component of CSI, we analyze and compare the SNR and error performance of these methods using a few feedback bits.

A natural drawback of the multiple-antenna systems is the increased cost and complexity due to the need for multiple radio-frequency (RF) chains, as a conventional multiple-antenna system requires the number of RF chains to be equal to the total number of antennas. Therefore, a scheme with small number of antennas at the mobile set and simple receiver complexity as well as a reduced number of RF chains is desirable for downlink transmission. However, most of the limited feedback transmit diversity schemes [8–10] require relatively high computational complexities due to vector quantization of beamforming vectors. For this reason, we focus on simple and practical limited feedback transmit diversity schemes and propose the hybrid method which reduces the transmitter complexity and uses the feedback information more efficiently compared to PPC method. We also propose a fast almost-optimal procedure to find the PPC codewords which significantly reduces the complexity. To improve the performance of the simple and practical antenna selection method in correlated fading channels and also Rician fading channels, we present the FFT-based antenna selection scheme which substantially outperforms the antenna selection under such channel conditions.

In closed-loop transmit diversity schemes, the channel information is provided to the transmitter through the feedback channel by the receiver. In real systems, this channel information is extracted by estimating the channel response at the receiver and quantizing it and sending it back through feedback channels. Therefore the channel estimation errors, the quantization errors and the feedback delay inevitably affect the performance of closed-loop transmit diversity schemes. Thus it is practically and theoretically important to quantify the performance degradation of the practical limited-feedback transmit diversity schemes in the presence of such channel errors. Few published analytical results exist for the effect of channel errors on the performance of limited-feedback transmit diversity schemes and most of them consider only one kind of channel error [11–13]. In this thesis, we analyze the impact of channel estimation errors as well as feedback delay on the performance of PPC, hybrid and transmit antenna selection schemes.

1.2 Contributions

The contributions of this thesis are as follows:

1. A practical hybrid scheme is proposed. For 2-transmit antenna systems this method adjusts the power of the transmitted signals using one-bit magnitude information along with steering the phase of the transmitted signals using the phase information of subchannels such that the received SNR is maximized. In this case, the proposed method outperforms the PPC method and performs very closely to the optimal beamforming scheme (OBS) using very few number of feedback bits (for example, at the BER of 10^{-3}, the proposed hybrid method has 0.50 dB gain over the PPC and only 0.15 dB power penalty compared to the OBS in BPSK systems). For more than 2 transmit antennas systems, this scheme performs phase steering on a selected subset of the strongest subchannels; thus it requires considerably reduced electronic circuitry and less computational complexity at the transmitter compared with the PPC and equal gain transmission (EGT) methods. This scheme results in performance improvement by avoiding the energy waste over the insignificant subchannels.

2. We propose a fast almost-optimal procedure for the optimization of the PPC codewords. This procedure reduces the complexity of the optimization of discrete PPC coefficients from the order of $O\left(2^{m(N-1)}\right)$ (required by exhaustive search) to the order of $O\left(N-1\right)$, where N is the number of transmit antennas and m is the number of feedback bits on the phase of each channel coefficient. We also derive the average SNR of the intuitive PPC method in N-transmit antenna systems.

3. We propose the transform domain selection method and as a practical example of it, the FFT-based antenna selection method (FFT method) which applies Fast Fourier Transformation (FFT) on the channel vector before extracting the feedback information. The column of the FFT matrix corresponding to the selected transformed subchannel is applied as beamformer at the transmitter. We show that the FFT method substantially outperforms the antenna selection method in correlated fading channels and Racian fading channels with a linear array of omnidirectional antennas.

4. We derive the exact average SNR of PPC method in the presence of channel estimation errors and feedback delay. In this regard, we derive the pdf of the phase difference between the estimated and predicted channel coefficients.

5. We analyze the performance of the hybrid method in the presence of channel estimation errors and feedback delay and adapt the hybrid weighting coefficients with the statistics of the channel errors for 2×1 systems. We derive the average SNR of hybrid method with feedback delay in two cases where the correlation coefficient between the estimated and predicted channel coefficients are known or unknown to the transmitter.

6. We derive the exact average SNR and average BER expressions for transmit antenna selection method under channel estimation errors and feedback delay in 2×1 systems. We analyze the combined and separate impact of channel estimation errors and feedback delay on the diversity gain and coding gain of this method.

7. We show that the channel estimation errors does not change the diversity gain

compared to the perfect channel estimation case and the performance degradation is due to reduction of coding gain by an identical factor for all methods, when the channel estimation error decreases with SNR. In this case we also show that feedback delay results in severe performance degradation by reducing the diversity gain as low as one (the same as SISO case).

1.3 Organization of Thesis

We proceed by introducing transmit diversity schemes, presenting our proposed schemes and analyzing and comparing their performances in the Chapter 2. This chapter begins with introduction and literature review of transmit diversity schemes in Section 2.1 followed by the description of the system model in Section 2.2. Section 2.3 introduces several transmit diversity schemes: The optimal beamforming scheme (OBS) and Alamouti scheme are introduced in Sections 2.3.1 and 2.3.2. The transmit antenna selection scheme is described in Section 2.3.3 where its average SNR and BER performances are analyzed. The two approaches for PPC method and our proposed fast almost-optimal procedure for optimizing the PPC coefficients are presented in Section 2.3.4. Our proposed hybrid method is presented separately for 2-transmit antennas and also N-transmit antennas MISO systems in Section 2.3.5. The transform domain selection method and specifically the proposed FFT method are presented in Section 2.3.6. The performance comparisons and numerical results are provided in Section 2.4 and correlated Rayleigh followed by the summary and conclusions of the chapter in Section 2.5.

Chapter 3 analyzes the impact of channel estimation errors and feedback delay on the performance of the PPC, Hybrid and transmit antenna selection methods. The introduction and the system model are presented in Sections 3.1 and 3.2 respectively. The performance of the PPC method in the presence of feedback delays is analyzed and the exact average SNR of this method with feedback delay is derived in Section 3.3. The impact of channel estimation errors and feedback delay on the performance of hybrid method in 2-transmit antenna systems with and without the knowledge of channel errors is studied in Section 3.4 where the average SNR is derived in these two cases. The performance of transmit antenna selection scheme under channel estimation errors and feedback delay is also analyzed and the exact closed-form average SNR and BER expressions for this method are derived in Section 3.5 for 2×1 systems. The evaluation and numerical results as well as performance comparisons are presented in Section 3.6.

At last, Chapter 4 concludes this thesis and suggests possible future research directions.

Chapter 2

Simple Limited Feedback Transmit Diversity Schemes

2.1 Introduction

To overcome the impact of fading which causes performance loss in wireless communications systems, diversity techniques are proposed to decrease the probability of having an overall weak channel. In general, diversity means using different dimensions of the channel, e.g. space, time, frequency, and so on, to improve the equivalent channel seen by the receiver. The spatial diversity refers to the existence of several paths that are sufficiently separated in space and fade almost independently and it is unlikely that they fade together. The spatial diversity is introduced by the usage of multiple antennas at the receiver (receive diversity), the transmitter (transmit diversity) or at both of them.

The receive diversity is applicable in the base station receivers (uplink) and due to implementation complexities and size limitation of the mobile terminal it is not generally practical for downlink applications. For downlink applications, multiple antennas can be used at the transmitter instead of the receiver to obtain spatial diversity. In this thesis we concentrate on multiple-input singe-output (MISO) wireless systems employing the transmit antenna diversity. As a simple and practical example of MISO systems we also study 2×1 systems since two transmit antenna base-stations have already been accepted as part of the third generation wireless standard and most mobile units can only use a single antenna. Schemes using two transmit antennas, such as space-time transmit diversity, spacetime spreading, and transmit antenna array, have been included in the the downlink of the Universal Mobile Telecommunications System (UMTS) [14] and IS-2000 standard [15].

Transmit antenna diversity schemes as means of improving the base station to mobile performance can be generally classified into closed-loop (some CSI at the transmitter) and open-loop (no CSI at the transmitter). Space-time codes [16, 17] such as orthogonal space-time block codes (OSTBC's) [18] and Alamouti code [19] are the well-known examples of open-loop systems, which are used in third generation cellular networks [20]. The open-loop transmit diversity schemes operate without any feedback information from the mobile and are known to offer diversity gain which brings about performance improvement. The closed-loop transmit diversity schemes (e.g. [8, 9, 21, 22]) which operate with feedback information from the mobile, offer not only diversity gain but also coding gain. Therefore, they offer more performance improvement compared to traditional open-loop transmit diversity schemes. For example, the OSTBC's SNR loss compared to optimal beamforming (with perfect CSI) is of the order of $10\log_{10}(NR_s)$ dB, where N is the number of transmit antenna elements and R_s is the code rate [18].

The closed-loop transmit diversity techniques adapt the transmitted signal to the channel conditions by applying some form of beamforming. The optimal beamforming which fully adapts the transmitted signal to the channel state and achieves the maximum SNR, requires perfect CSI at the transmitter. In practice, the feedback link has bandwidth constraint and complete CSI is not available at the transmitter. In such cases, one approach for obtaining CSI at the transmitter is to quantize the channel response at the receiver and to send it to the transmitter through a low bandwidth feedback channel [5, 8–10, 21–23]. To support the limitations of the feedback channel, the closed-loop beamforming methods work by using a codebook of beamforming vectors that is designed off-line and is fixed for all channel realizations and is known at both the transmitter and the receiver. The receiver is assumed to convey the best beamforming vector from the codebook over an error-free, zero-delay feedback channel. The codebook design criteria include maximizing the average SNR at the maximum ratio combining (MRC) output [5, 10], maximizing the average mutual information [24–27], or minimizing the outage probability [8].

In general, the codebook design can be viewed as a vector quantization (VQ) problem, and the generalized Lloyd algorithm can be employed to actually construct the codebook. The primary work on quantized beamforming in [5] was intended to quantize the optimal beamforming scheme (OBS) [28] using the Lloyd algorithm to obtain the beamforming codebooks and did not develop a specific codebook design methodology. The quantization of the equal gain transmission (EGT) was proposed in [7] as the partial phase combining (PPC) method which uniformly quantized the phases of the channel. Different codebooks have been designed in [29] and [30], extending the work in [7]. Specialized variations of quantized OBS and PPC for two transmit antennas are part of the wide-band code division multiple access (WCDMA) closed-loop diversity mode [20].

The vector quantization problem has been cast as an equivalent sphere vector quantization (SVQ) problem in [9] using the fact that in i.i.d. Rayleigh fading channels designing beamformer codebooks reduces to a SVQ problem, where the codewords are vectors constrained on the unit hypersphere and the vector source input is uniformly distributed on the unit hypersphere. The Grassmannian beamforming is proposed in [10], where a beamforming design criterion is derived based on thinking of the codebook vectors as points in the Grassmannian manifold. Most of the discussed methods do not specifically address the design of hardware-constrained beamformers. Imposing additional constraints on the beamforming vector codebook, such as equal gain coefficients or selection columns, makes limited feedback beamforming more practical than beamforming with arbitrary codebooks (e.g., in [20]). One major drawback of the discussed methods which strive to obtain near-optimal codebook designs is their high computational complexity and hardware cost due to jointly quantizing (vector quantization) of the parameters of the channel. For this reason, we consider quantizing each parameter independently (scalar quantization) to achieve lower complexities and study and propose practical low-complexity transmit diversity schemes. By jointly quantizing the channel parameters the importance and impact of each parameter in different channel conditions is not distinguished.

In this chapter, we evaluate/compare the SNR and BER performances of several transmit diversity schemes for MISO systems and propose a hybrid method, an FFT-based selection method and an almost-optimal procedure for optimizing PPC coefficients. In order to gain insight to the value and efficiency of different kinds of

CSI we study the antenna selection method [6,31,32] and the PPC method [7] where the latter only applies the phase information and the former applies only the magnitude information of the channel. We also propose a simple practical hybrid method in which a subset of subchannels with stronger gains are first selected. Then, the PPC is performed over the signals of the selected subchannels. The hybrid method (compared with the PPC method) requires a cheaper transmitter (i.e., considerably reduced electronic circuitry and less computational complexity). We show that as the number of the feedback bits on the phase of subchannels increases, the hybrid scheme outperforms the PPC scheme with the same number of feedback bits and even the equal gain transmission (EGT) scheme which requires unlimited feedback bits. For 2-transmit antenna systems, the proposed hybrid method adapts the power of the transmitted signals based on the one bit information of subchannel magnitudes and adjusts their phase such that the received SNR is maximized. This method outperforms the PPC method with the same number of feedback bits and performs very close to the OBS when at least three feedback bits are available.

We extend the 2×1 partial phase combining (PPC) method [7] for MISO systems, and propose a fast almost-optimal procedure for determination of the feedback codeword in this method. We derive the average SNR expressions for the hybrid and the PPC methods. We also present the transform-domain selection method which applies an orthogonal transformation before extracting feedback bits. We use the fast Fourier Transform (FFT) as special case and show that in systems with correlated fading channels and systems with uniform linear array of omnidirectional antennas, this method substantially outperforms the antenna selection method. We apply a simple approach to derive the average bit error rate (BER) in general by applying the cumulative distribution function (cdf) of the received SNR and the definition of Gamma function. Using derived expressions and numerical results for the average SNR and BER we compare performance of the discussed transmit diversity schemes, which gives insight to the effect and value of allocating each bit of feedback to the magnitude and phase information of the channel.

The rest of this chapter is organized as follows. In Section 2.2, the system model is described. In Section 2.3, several simple transmit diversity schemes are presented for MISO systems including the proposed hybrid scheme and the transform domain selection scheme and average SNR and BER expressions are derived for some of them. Section 2.4 provides the performance evaluation and comparison of different schemes. Finally, Section 2.5 provides the summary and conclusions of the chapter.

2.2 System Model

We consider a single-user multiple antenna system with N transmit and one receive antennas. We assume uncorrelated Rayleigh flat-fading channel and additive white Gaussian noise. The received signal is given by:

$$r = h_1 s_1 + h_2 s_2 + \cdots + h_N s_N + n = \mathbf{h}\mathbf{s} + n, \tag{2.1}$$

where $\mathbf{h} = [h_1, h_2 \cdots, h_N]$ is the channel vector with channel coefficients $h_1, h_2 \cdots, h_N$, which are independent and identically distributed (i.i.d.) zero mean circularly symmetric complex Gaussian (CSCG) random variables with unit variance, and $\mathbf{s} = [s_1, ..., s_N]^T$ is the transmit vector and n represents the additive noise which is a zero-mean complex Gaussian random variable with variance $\sigma_n^2 = E[|n|^2]$. The total transmission power is $P = \sum_{k=1}^{N} E[|s_k|^2]$. In transmit beamforming schemes, the transmit vector $\mathbf{s}$ is attained by multiplying the transmit signal s by an $N \times 1$ normalized transmit beamforming vector, $\mathbf{v}$. In these schemes the received signal is given by,

$$r = \mathbf{hv}s + n. \tag{2.2}$$

2.3 Simple Transmit Diversity Schemes

In this Section we overview several simple transmit diversity schemes using perfect CSI, no CSI and partial CSI and present our proposed hybrid scheme, FFT-based transmit antenna selection scheme and also the almost-optimal procedure for the PPC scheme.

2.3.1 Optimal Beamforming Scheme

If perfect CSI is available at the transmitter, the optimal beamforming scheme (OBS) also known as maximum ratio transmission (MRT) can be used which achieves the maximum average SNR by multiplying the ith transmitted signal by the normalized conjugate of the ith channel coefficient. In other words, the beamformer vector $\mathbf{v} = \frac{\mathbf{h}^H}{||\mathbf{h}||}$ is applied at the transmitter, where $(.)^H$ denotes the conjugate transpose and $||.||$ represents the Euclidean vector norm. This scheme is illustrated schematically in Figure 2.1. The received signal in this method is,

$$r = \frac{\mathbf{hh}^H}{||\mathbf{h}||} s + n = ||\mathbf{h}|| s + n = \sqrt{\sum_{k=1}^{N} |h_k|^2} s + n. \tag{2.3}$$

The instantaneous SNR in this case is,

$$\text{SNR} = \frac{P}{\sigma_n^2} \sum_{k=1}^{N} |h_k|^2, \tag{2.4}$$

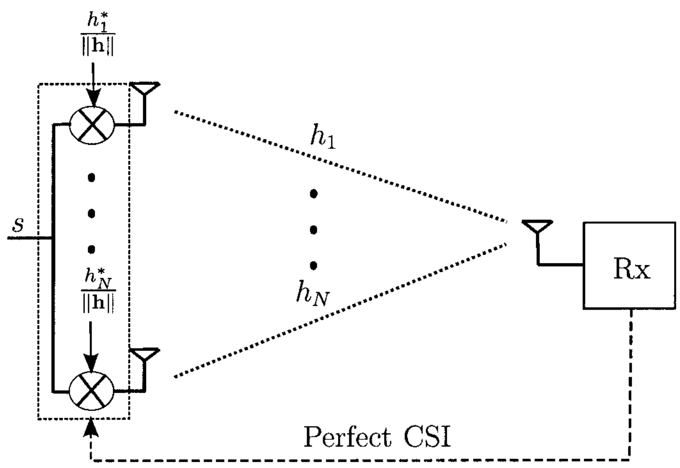

Figure 2.1: Block diagram of optimal beamforming scheme.

and the maximum achievable average SNR is

$$\overline{\text{SNR}} = E\left[\text{SNR}\right] = \frac{P}{\sigma_n^2} E\left[\sum_{k=1}^{N} |h_k|^2\right] = \frac{NP}{\sigma_n^2}, \tag{2.5}$$

using the fact that $E\left[|h_k|^2\right] = 1$ for all $k = 1, \cdots, N$. We call $\frac{P}{\sigma_n^2}$ as Transmit-power-to-Noise-Ratio (TNR). In Appendix F, we derive the exact closed-form BER expression for the OBS (using BPSK or QPSK with Gray coding) as,

$$P_e(\text{OBS}) = \frac{1}{2}\left(1 - \sum_{\text{i}=0}^{\text{N}-1} \frac{(2\text{i})!}{2^{\text{i}}(\text{i}!)^2} \frac{\sqrt{\beta}}{(\beta+2)^{\text{i}+\frac{1}{2}}}\right), \tag{2.6}$$

where $\beta = \frac{2P}{\sigma_n^2 \log_2 D}$ and D is the number of constellation points. Note that for other modulation schemes similar expressions could be derived as upper bound for the BER in the same way. This average BER expression is a special case of the result in [33]. For $N = 2$ the average BER in (2.6) reduces to:

$$P_e(\text{OBS}) = \frac{1}{2}\left(1 - \sqrt{\frac{\beta}{\beta+2}} - \frac{\sqrt{\beta}}{(\beta+2)^{\frac{3}{2}}}\right). \tag{2.7}$$

Using the average BER expression derived in (2.7) and Appendix I, we calculate the diversity gain, G_d, and coding gain, G_c, as $G_d = 2$ and $G_c = \sqrt{\frac{4}{3}}$ respectively.

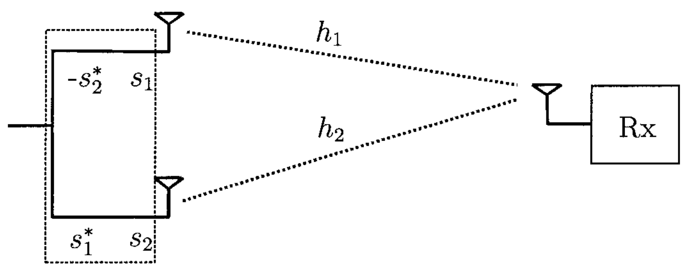

Figure 2.2: Alamouti scheme.

In many real systems, the CSI can not be fully provided to the transmitter, for example, due to a limited capacity of feedback channel or rapid channel variations. In these imperfect CSI cases, the achievable performance is bounded by that of the optimal beamforming scheme. The optimal beamforming transmitter is computationally complex and requires significant circuitry. Therefore, many designers are motivated to reduce the price of the transmitter which implies the partial/imperfect usage of the CSI at the transmitter.

2.3.2 Alamouti Scheme

In the Alamouti scheme, which is proposed by Alamouti in [19] for 2-transmit antenna systems, an orthogonal space time block coding is performed at the transmitter with no need for CSI at the transmitter. This scheme has now been incorporated in third generation cellular communication systems. Figure 2.2 demonstrates this method schematically. In this scheme the signals s_1 and s_2 are sent simultaneously from first and second antennas respectively during the first time interval, and signals $-s_2^*$ and s_1^* are transmitted during the second time interval. For 2×1 systems, the corresponding received signals in these two intervals can be expressed as,

$$\begin{aligned} r_1 &= h_1 s_1 + h_2 s_2 + n_1 \\ r_2 &= -h_1 s_2^* + h_2 s_1^* + n_2, \end{aligned} \tag{2.8}$$

where n_i represents the channel noise. In the Alamouti coding, the decoding of s_1 and s_2 is based on

$$\begin{aligned} \hat{s}_1 &= h_1^* r_1 + h_2 r_2^* \\ \hat{s}_2 &= h_2^* r_1 - h_1 r_2^*, \end{aligned} \tag{2.9}$$

The signals are detected using maximum-likelihood detection. When the signals are transmitted with the same power, $\frac{P}{2}$, the received SNR is given by,

$$\text{SNR} = \frac{P}{2\sigma_n^2}(|h_1|^2 + |h_2|^2). \tag{2.10}$$

As it is observed the SNR in the Alamouti scheme is half of the SNR in the OBS in (2.4) when $N = 2$ and therefore this method incurs 3dB power penalty compared to the OBS. The average SNR in this scheme is

$$\overline{\text{SNR}} = \frac{P}{2\sigma_n^2} E\left[|h_1|^2 + |h_2|^2\right] = \frac{P}{\sigma_n^2}, \tag{2.11}$$

since $E\left[|h_1|^2\right] = E\left[|h_2|^2\right] = 1$. As the SNR in Alamouti scheme is half of the SNR in OBS, the exact closed form BER expression for this scheme is easily found from the BER expression of OBS by replacing β by $\frac{\beta}{2}$ and using $N = 2$ in (2.6),

$$P_e(\text{Alamouti}) = \frac{1}{2}\left(1 - \sqrt{\frac{\beta}{\beta+4}} - 2\frac{\sqrt{\beta}}{(\beta+4)^{\frac{3}{2}}}\right). \tag{2.12}$$

2.3.3 Transmit Antenna Selection Method

One major drawback of multiple-antenna systems is the need for multiple radio-frequency (RF) chains (equal to the total number of antennas), which leads to high

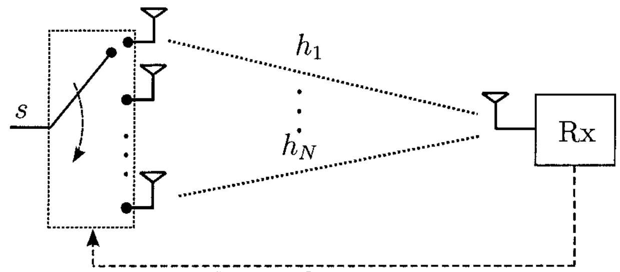

Figure 2.3: Transmit antenna selection scheme.

implementation costs as the RF chains are usually the most expensive part of a MIMO system. A promising approach for reducing implementation costs and complexity, while retaining a reasonably good performance, is to employ some form of antenna selection [6, 34]. The antenna selection schemes optimally choose a subset of the available transmit and/or receive antennas according to some selection criterion, and process the signals associated with these selected antennas. The feedback required by antenna selection is only a small fraction of the full channel state information.

Here we study the single transmit antenna selection scheme with the selection criterion of maximizing the received SNR. Figure 2.3 provides schematic demonstration of this scheme. In this scheme, the index of the channel coefficient with the largest magnitude is conveyed from the receiver to the transmitter with $\lceil \log_2 N \rceil$ number of bits, where $\lceil a \rceil$ denotes the smallest integer bigger than or equal to a. The transmitter utilizes this information in order to increase SNR by transmitting signal only from the selected subchannel. The main advantage of this method is the simplicity of the transmitter. The received signal in this scheme is given by,

$$r = h_{\arg\max_{i=1,\ldots,N}\{|h_i|\}} s + n. \tag{2.13}$$

and the received SNR is,

$$\mathrm{SNR} == \frac{P}{\sigma_n^2} \max_{i=1,\ldots,N} \left\{|h_i|^2\right\}. \tag{2.14}$$

In Appendix A, we calculated the average SNR in this scheme as follows,

$$\overline{\mathrm{SNR}} = \frac{P}{\sigma_n^2} E\left[\max_{i=1,\ldots,N} |h_i|^2\right] = \frac{P}{\sigma_n^2} \sum_{k=0}^{N-1} \binom{N}{k+1} \frac{(-1)^k}{(k+1)}. \tag{2.15}$$

In the 2×1 system where only one bit feedback is used for the antenna selection, the average SNR in (2.15) becomes, $\overline{\mathrm{SNR}} = 1.5\frac{P}{\sigma_n^2}$ which shows $10 \log 1.5 = 1.76$dB gain over the Alamouti scheme and $10 \log \frac{2}{1.5} = 1.24$dB loss compared to OBS.

In Appendix E, we derive the exact closed-form BER expression for this method (using BPSK or QPSK with Gray coding) as,

$$P_e = \frac{1}{2} \sum_{i=0}^{N} (-1)^i \binom{N}{i} \sqrt{\frac{\beta}{\beta + 2i}}. \tag{2.16}$$

The above expression is a special case of the result in [35]. Using the average BER expression derived in (2.16) and Appendix I, we find that in the antenna selection method the diversity gain is $\left(\frac{(-2)^N (N-1)!}{(2N-1)!}\right)^{\frac{1}{N}}$ and coding gain is N. For $N = 2$ the average BER is,

$$P_e = \frac{1}{2}\left(1 - 2\sqrt{\frac{\beta}{\beta+2}} + \sqrt{\frac{\beta}{\beta+4}}\right). \tag{2.17}$$

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2.3.4 Partial Phase Combining Method

The partial phase combing (PPC) method was first introduced by Heath and Paulraj in [7] and can be viewed as approximate equal gain combining (EGC) method which fully steers the phases of the transmit signals. Figure 2.4 provides schematic demonstration of the PPC method. We present two ways to perform the PPC and propose a

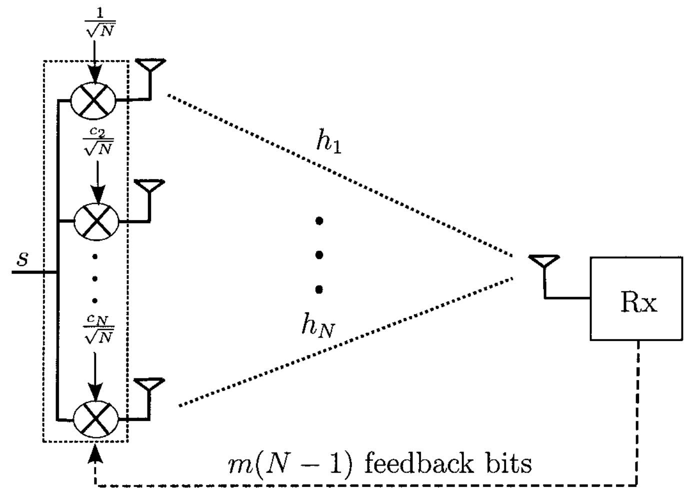


Figure 2.4: Partial phase combining (PPC) scheme.

fast almost-optimal algorithm for determination of PPC codewords in a MISO system. We also derive the exact closed-form average SNR in this method.

PPC-I

In this method, the phase of the transmitted signals over different antennas are adjusted such that they are received synchronized with the received signal from the first antenna element. The phase difference between each channel coefficient and the first one is quantized and sent back to the transmitter using m bits. The transmitter makes a phase shift of $Q(\angle h_1 - \angle h_i)$ to the signal of the ith subchannel such that the signals are received synchronized at the receiver, where $Q(\angle h_1 - \angle h_i)$ is the quantized phase difference between h_1 and h_i and $Q(.)$ represents the uniform quantizer. For instance, the signals $s_1 = \frac{s}{\sqrt{N}}, s_2 = \frac{s}{\sqrt{N}} e^{jQ(\angle h_1 - \angle h_2)}, \ldots, s_N = \frac{s}{\sqrt{N}} e^{jQ(\angle h_1 - \angle h_N)}$ are transmitted respectively with the same power $P_i = \frac{P}{N}$ which is equivalent to applying the beamforming vector $\mathbf{v} \triangleq \frac{1}{\sqrt{N}} [1, c_2 \ldots, c_N]^T = \frac{1}{\sqrt{N}} [1, e^{jQ(\angle h_1 - \angle h_2)}, \ldots, e^{jQ(\angle h_1 - \angle h_N)}]^T$. In this case, the received signal becomes,

$$r = \left(h_1 + h_2 e^{jQ(\angle h_1 - \angle h_2)} + \ldots + h_N e^{jQ(\angle h_1 - \angle h_N)}\right) \frac{s}{\sqrt{N}} + n. \tag{2.18}$$

We calculated the average SNR in this method in Appendix B as follows

$$\overline{\text{SNR}} = \frac{P}{N\sigma_n^2} \text{E}\left[\left|\sum_{i=1}^{N} h_i e^{jQ(\angle h_1 - \angle h_i)}\right|^2\right] \tag{2.19}$$

$$= \frac{P}{N\sigma_n^2}\left[N + (N-1)2^{m-1}\sin\frac{\pi}{2^m} + (N-1)(N-2)\frac{2^{2m-2}}{\pi}\sin^2\frac{\pi}{2^m}\right] \tag{2.20}$$

For the special case when $m = 1$, the average SNR is:

$$\overline{\text{SNR}} = \frac{P}{N\sigma_n^2}\left(2N - 1 + \frac{(N-1)(N-2)}{\pi}\right). \tag{2.21}$$

In a 2×1 system the average SNR in (2.20) reduces to

$$\overline{\text{SNR}} = \frac{P}{\sigma_n^2}\left(1 + 2^{m-2}\sin\frac{\pi}{2^m}\right). \tag{2.22}$$

We observe from (2.22) that in a 2×1 system the one-bit feedback PPC (i.e. $m = 1$) and the antenna selection schemes result in the same average SNR of $1.5\frac{P}{\sigma_n^2}$, which is 1.76dB higher than Alamouti scheme. This is justified by the fact that using one-bit feedback the PPC reduces to finding $\max\{\frac{|h_1+h_2|^2}{2}, \frac{|h_1-h_2|^2}{2}\}$ which has the same distribution as $\max\{|h_1|^2, |h_2|^2\}$. Therefore, both the antenna selection and 1-bit PPC schemes have the same received SNR distribution, the same average SNR and the same average BER expressions in Rayleigh fading channels. Thus, allocating one bit of feedback to either the phase or the magnitude comparison of channel coefficients results in the same performance in Rayleigh fading channels. In a fading channel if we know that the subchannels are heavily correlated $|h_1| \approx |h_2|$, the antenna selection method provides insignificant SNR improvement over the Alamouti's scheme, therefore, it is definitely more gainful to use PPC method. This case happens for instance, if there is an strong line-of-sight (LOS) between transmitter and the receiver. We also observe from (2.22) that in a 2×1 system by using the PPC method with two feedback bits (i.e. $m = 2$), the average SNR becomes 2.32dB higher than that of Alamouti's scheme and only 0.67dB away from maximum achievable average SNR using OBS. For a large number of feedback bits, $m \to \infty$, this method performs as close as 0.49dB in average SNR to the OBS.

If the number of feedback bits used in the PPC scheme is sufficiently large the transmitter fully steers the transmitted signals over different antenna elements toward the receiver at the same energy level. This asymptotic case can be viewed as the equal gain transmission (EGT) [30]. In this asymptotic case where $m \to \infty$ the average SNR in (2.20) converges to,

$$\overline{\text{SNR}} = \frac{P}{N\sigma_n^2}\left[N + \frac{\pi}{2}(N-1) + \frac{\pi}{4}(N-1)(N-2)\right] \tag{2.23}$$

PPC-II (Using the Proposed Procedure)

The previous method intuitively synchronizes the received signal components yielding the performance improvement which is not optimal. In the second method, we attempt to optimize the PPC beamforming vector $\mathbf{v} \triangleq \frac{1}{\sqrt{N}}\left[1, c_2 \ldots, \; c_N\right]^T$, with $c_2, \ldots, c_N \in \mathcal{C} = \{1, e^{j\frac{\pi}{2^{m-1}}}, \ldots, e^{j\frac{(2^m-1)\pi}{2^{m-1}}}\}$, such that the average SNR is maximized. The average SNR in this method is given by,

$$\overline{\text{SNR}} = \max_{c_2,\ldots,c_N \in \mathcal{C}} \left(E\left[\left|h_1 + c_2 h_2 + \cdots + c_N h_N\right|^2\right] \frac{P}{N\sigma^2}\right). \tag{2.24}$$

The PPC vector that maximizes the above is conveyed to the transmitter using $m(N-1)$ feedback bits. In general, the optimization of discrete values $c_2, \ldots, c_N \in \mathcal{C}$ requires exhaustive search with complexity order of $O\left(2^{m(N-1)}\right)$. We propose a fast almost-optimal procedure to find the coefficients with complexity order of $O\left(N-1\right)$. The procedure is as follows, Step 1: $S_1 = h_1$,

Step 2: for $i = 2, \ldots, N$ choose c_i as follows:

$$\begin{aligned} c_i &\triangleq \arg\max_{c_i \in \mathcal{C}} |S_{i-1} + c_i h_i| \Rightarrow \angle c_i = Q(\angle h_i - \angle S_{i-1}), \\ S_i &= S_{i-1} + c_i h_i \end{aligned} \tag{2.25}$$

Simulation results in Section 2.4 show that for $m \geq 4$, the almost-optimal PPC performs the almost same as the EGT by using far fewer feedback bits and having much less computational complexity. For small values of m, this method significantly outperforms the PPC-I method with the price of only $N - 1$ extra additions.

2.3.5 Proposed Hybrid Method

In this proposed method the information about both the phase and the magnitude of channel coefficients is sent back to transmitter. We propose two schemes for the cases of $N = 2$ and $N > 2$ as follows:

Hybrid method for $N = 2$

In this scheme one bit of feedback information determines the subchannel with larger coefficient magnitude and m bits are assigned to the phase difference of channel coefficients. Based on the 1-bit magnitude information, transmitted signals are multiplied by weighting coefficients g_1 and g_2 and based on the m bits information of quantized phase difference, phase shift is applied on the second subchannel's signal. The schematic demonstration of this scheme is provided in Figure 2.5. The received signal is given by:

$$r = \left(g_1 h_1 + g_2 h_2 e^{jQ(\angle h_1 - \angle h_2)}\right) s + n, \tag{2.26}$$

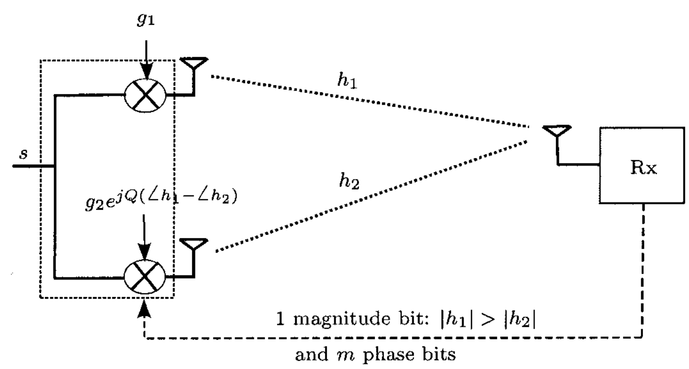


Figure 2.5: The proposed hybrid scheme for two-transmit antenna systems.

where g_1 and g_2 are positive real numbers satisfying $g_1^2 + g_1^2 = 1$ to keep total transmission power fixed. The weighting coefficients g_1 and g_2 are chosen such that average SNR is maximized given which channel coefficient has the largest magnitude;

$$g_1, g_2 = \arg \max_{g_1,\ g_2 \in \mathbb{R}^+,\ g_1^2+g_1^2=1} E\left[\left|g_1 h_1 + g_2 h_2 e^{jQ(\angle h_1 - \angle h_2)}\right|^2 \Big| |h_1| > |h_2|\right]. \quad (2.27)$$

In Appendix C, g_1 and g_2 are found as,

$$g_1 = \sqrt{\frac{1}{2}\left(1 + \frac{1}{\sqrt{1 + 2^{2m-2}\sin^2\frac{\pi}{2^m}}}\right)}, \quad (2.28)$$

$$g_2 = \sqrt{\frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + 2^{2m-2}\sin^2\frac{\pi}{2^m}}}\right)}. \quad (2.29)$$

The average SNR in this scheme is derived in Appendix C as,

$$\overline{\mathrm{SNR}} = \frac{P}{\sigma_n^2}\left(\frac{3}{2}g_1^2 + \frac{1}{2}g_2^2 + g_1 g_2 2^{m-1}\sin\frac{\pi}{2^m}\right) \quad (2.30)$$

$$= \frac{P}{\sigma_n^2}\left[1 + \frac{1}{2}\sqrt{1 + 2^{2m-2}\sin^2\frac{\pi}{2^m}}\right]. \quad (2.31)$$

For example if the phase difference is quantized with one bit, ($m = 1$), g_1 and g_2 are 0.9239 and 0.3827, respectively and the average SNR is $1.71\frac{P}{\sigma_n^2}$ which represents 2.32dB improvement over that of Alamouti scheme and is only 0.67dB away from average SNR of OBS. We observe that in Rayleigh fading channels, investing one bit of feedback on magnitude comparison and one bit on phase difference of channel coefficients attains the same SNR performance as assigning all the two feedback bits to their phase difference. For $m \geq 4$, the average SNR of hybrid scheme approaches $1.93\frac{P}{\sigma_n^2}$ which is as close as 0.14dB to average SNR attained by the OBS. We observe that by using just one-bit information about magnitude of subchannels we attain significant gains over PPC and close performance to OBS. This implies that having more than one-bit on magnitude information does not improve the performance significantly since using only one-bit on magnitude boosts up the performance very close to the OBS and leaves less performance gap with OBS to be improved by more bits on magnitude. The proposed hybrid scheme is simple to implement and allows better use of resources (the feedback bits) compared to the PPC and antenna selection methods.

Hybrid method for $N > 2$

For $N > 2$, M subchannels with stronger magnitudes are selected at the receiver using fast and efficient algorithms [36–38]. Then, the phases of the selected subchannels are quantized in the same way as the PPC-II method with m bits per subchannel. This information is sent back to the transmitter using $(N-1)+m(M-1)$ bits, since indicating the selected subchannels requires at most $N-1$ bits. The transmitter utilizes this information to transmit signal only from the selected subchannels and steer the phases of signals sent through them toward the receiver. Figure 2.6 illustrates this method schematically. As the hybrid method applies PPC only on the selected

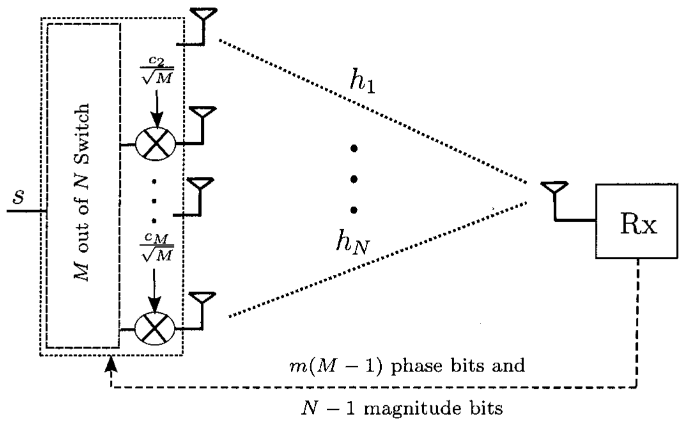


Figure 2.6: The proposed hybrid scheme for N-transmit antenna systems.

subchannels, it requires considerably reduced electronic circuitry and less computational complexity at the transmitter compared with the PPC and EGT methods. This method results in performance improvement by avoiding the energy waste over the insignificant subchannels.

2.3.6 Transform Domain Selection Method

In this method, an orthogonal transformation (OT) is first performed on the channel vector $\mathbf{h}$ at the receiver as $\mathrm{OT}(\mathbf{h}) = \mathbf{Th}$, where $\mathbf{T}$ is the orthogonal transformation matrix. Then some information bits about $\mathrm{OT}(\mathbf{h})$ is sent to the transmitter using, for example, one of the above methods and taking $\mathrm{OT}(\mathbf{h})$ as the channel vector. The transmitter performs similarly with an additional pre transformation of T^H. For instance, we can apply the antenna selection method on $\mathrm{OT}(\mathbf{h})$, i.e., the element of $\mathrm{OT}(\mathbf{h})$ with the largest magnitude is found at the receiver and its corresponding index, p_o, is conveyed to the transmitter using $\lceil \log_2 N \rceil$ feedback bits.

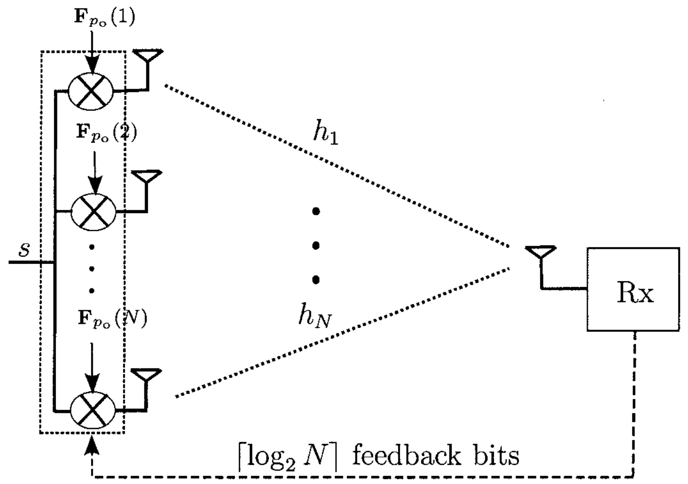


Figure 2.7: The FFT method.

We choose the fast Fourier Transformation (FFT) which is an orthogonal transformation with the advantage of a low computational complexity as a practical example. The FFT is performed on the channel vector:

$$\mathrm{FFT}(\mathbf{h}) = \mathbf{Fh}, \quad [\mathbf{F}]_{p,q} \triangleq \frac{e^{-j\frac{2\pi}{N}(p-1)(q-1)}}{\sqrt{N}}, \quad p, q = 1, \ldots, N. \tag{2.32}$$

The corresponding index of the element of $\mathrm{FFT}(\mathbf{h})$ with the largest magnitude (i.e., which maximizes the SNR) is found at the receiver as

$$p_o = \arg \max_{p\in\{1,\ldots,N\}} \left| \sum_{k=1}^{N} h_k e^{-j\frac{2\pi(k-1)(p-1)}{N}} \right|, \tag{2.33}$$

and sent back to the transmitter using $\lceil \log_2 N \rceil$ feedback bits. The transmitter employs the p_oth column of $\mathbf{F}$, i.e.,

$$\mathbf{F}_{p_o} \triangleq \frac{1}{\sqrt{N}}[1, e^{\frac{-j2\pi(p_o-1)}{N}}, \ldots, e^{\frac{-j2\pi(p_o-1)}{N}(N-1)}]^T,$$

as the beamforming vector and transmits $\mathbf{F}_{p_o}s$. The schematic demonstration of this scheme is provided in Figure 2.7. The received signal in this scheme is

$$r = \left(\frac{1}{\sqrt{N}} \sum_{k=1}^{N} h_k e^{-j\frac{2\pi}{N}(k-1)(p_o-1)} \right) s + n, \tag{2.34}$$

and the average SNR is given by,

$$\overline{\text{SNR}} = \frac{P}{N\sigma^2} E\left[\left|\sum_{k=1}^{N} h_k e^{-j\frac{2\pi}{N}(k-1)(p_o-1)}\right|^2\right]. \tag{2.35}$$

The average BER and SNR performance comparisons in Section 2.4 show that the FFT-based antenna selection method (FFT method) performs exactly the same as the antenna selection method in i.i.d. Rayleigh fading channels. The FFT method is advantageous over the antenna section method if the channel vector is not a zero mean i.i.d complex Gaussian multivariate. For example, in a system with LOS (i.e., $E[h_i] \neq 0$) and using linear array of omnidirectional antennas, we show that this method takes the advantage of the array signature. The signature of a uniform linear array of omnidirectional antennas represents a sinusoidal vector (with respect to the propagation direction). Therefore, the energy of the channel response is more compact in frequency domain, hence, the FFT makes the energy to become more concentrated in one of the components. As known in the literature of transform source coding [39], this consequently leads to significant performance improvement. However, we show that if the employed antennas are non-omnidirectional, the antenna selection method may outperform the FFT.

The antenna selection scheme works well for uncorrelated MIMO channels. However, in practice the transmitter (the base station) encounters correlated fading channels as it is typically placed high above the ground and sees no local scatters. Our simulation results in Section 2.4 reveal that in correlated fading channels, the FFT method outperforms the antenna selection method significantly and the more correlated the channel is, the more advantageous the FFT method would be. The suggested FFT method provides better performance compared to the antenna selection scheme for a wide range of channel configurations by employing the same number of feedback bits and having a very simple implementation.

2.4 Performance Comparison

Using the closed form average BER and SNR expressions derived in Appendices A, B, C, E, F and numerical results, we compare the BER and SNR performance of the discussed transmit diversity methods in systems with BPSK modulation for simplicity and without loss of generality.

Figures 2.8 (a) and (b) show the average BER curves versus the Transmit-power-to-Noise-Ratio (TNR) in a 2×1 system with uncorrelated Rayleigh fading channel. Figure 2.8 (a) shows that the antenna selection and one-bit PPC schemes have identical average BER performance and achieve 1.51dB TNR gain over Alamouti scheme and 1.49dB TNR penalty compared to OBS at the average BER of 10^{-3}. This implies that assigning only one bit feedback to either the phase or magnitude information results in the same performance in Rayleigh fading channels. We also observe that the BER performance of hybrid scheme with $m = 1$ is identical to that of the PPC method with $m = 2$. It can be deduced that assigning two feedback bits to phase information results in the same performance achieved by assigning one bit feedback to phase and one bit to magnitude information. Another important observation is that all the discussed methods (except SISO case) have almost parallel BER curves which implies that they have the same diversity gain (the negative slope of the average BER curves). This justifies the application of average SNR for performance comparison of these methods. We observe in Figure 2.8 (b) that the hybrid scheme with $m = 2$ performs very closely to the OBS by using only three bits of feedback. This scheme also significantly outperforms the PPC method that uses the same number of feedback bits. For example, at the average BER of 10^{-3} the hybrid scheme with $m = 2$ performs as close as 0.29dB to OBS and 0.39dB better than the PPC with $m = 3$.

Figure 2.9 depicts the required transmit power to noise ratio (TNR) versus the required numbers of feedback bits in order to achieve the average BER of 10^{-3}. This figure illustrates that as the number of feedback bits increases, the BER performance of the hybrid method becomes very close to the OBS (as close as 0.15dB in required TNR to achieve average BER of 10^{-3}). We observe that in the hybrid and PPC methods, using four bits of feedback achieves almost the same performance as using infinite bits. By using three feedback bits and more, the hybrid method outperforms PPC method with about 0.50dB higher gain in SNR.

Figures 2.10 (a) and (b) depict the average BER curves versus the TNR, $\frac{P}{\sigma^2}$, for a 5×1 system with an uncorrelated Rayleigh fading channel. It is observed that the FFT and the antenna selection methods have the same performance in a Rayleigh fading channel and require the least number of feedback bits among the discussed methods, i.e. $\lceil \log_2 N \rceil = 3$bits. At the average BER of 10^{-4}, the performances of these two methods are 4.5 dB away in TNR from the performance of the optimal beamforming method.

We observe that the hybrid method outperforms the PPC methods using the same total number of feedback bits, if the number of feedback bits assigned to each quantized phase difference is $m \geq 2$. Figure 2.10 (a) shows that at the average BER of 10^{-4} the hybrid method achieves 0.42 dB and 0.77 dB gain in TNR over the PPC-II and PPC-I methods respectively. We can also observe that for $m = 2$, the PPC-II method has 0.35 dB gain in TNR over the PPC-I method at the average BER of 10^{-4}.

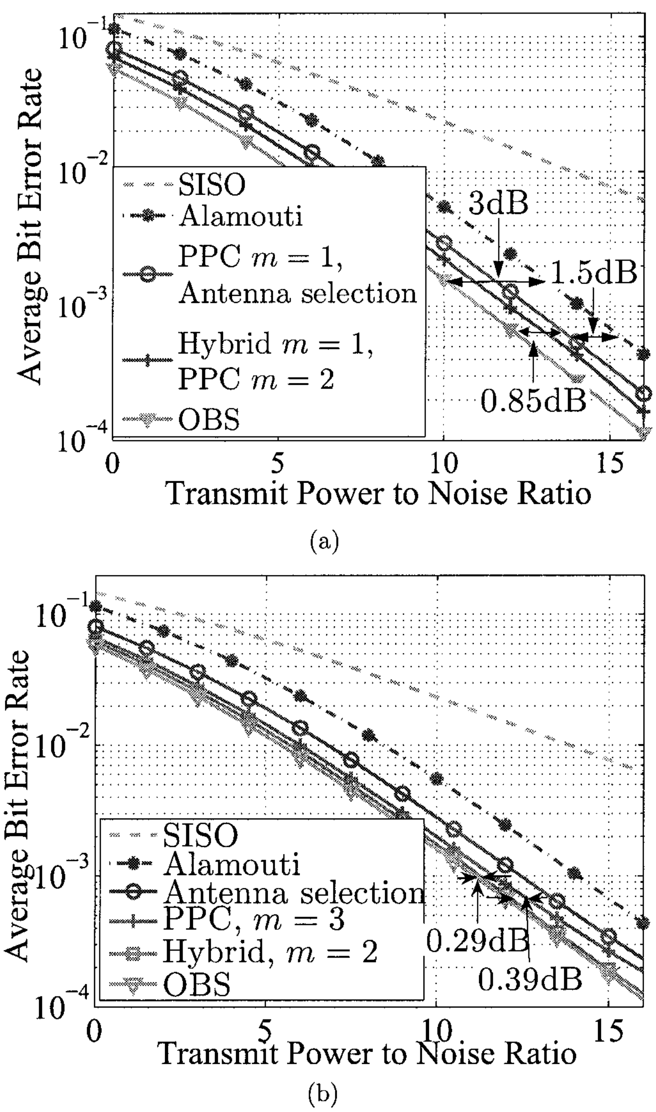


Figure 2.8: Average BER curves versus the required Transmit-power-to-Noise-Ratio (TNR) in dB for a 2×1 BPSK system.

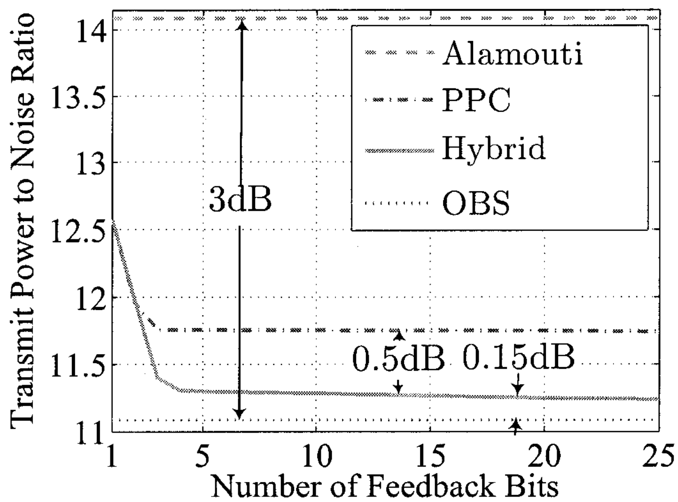
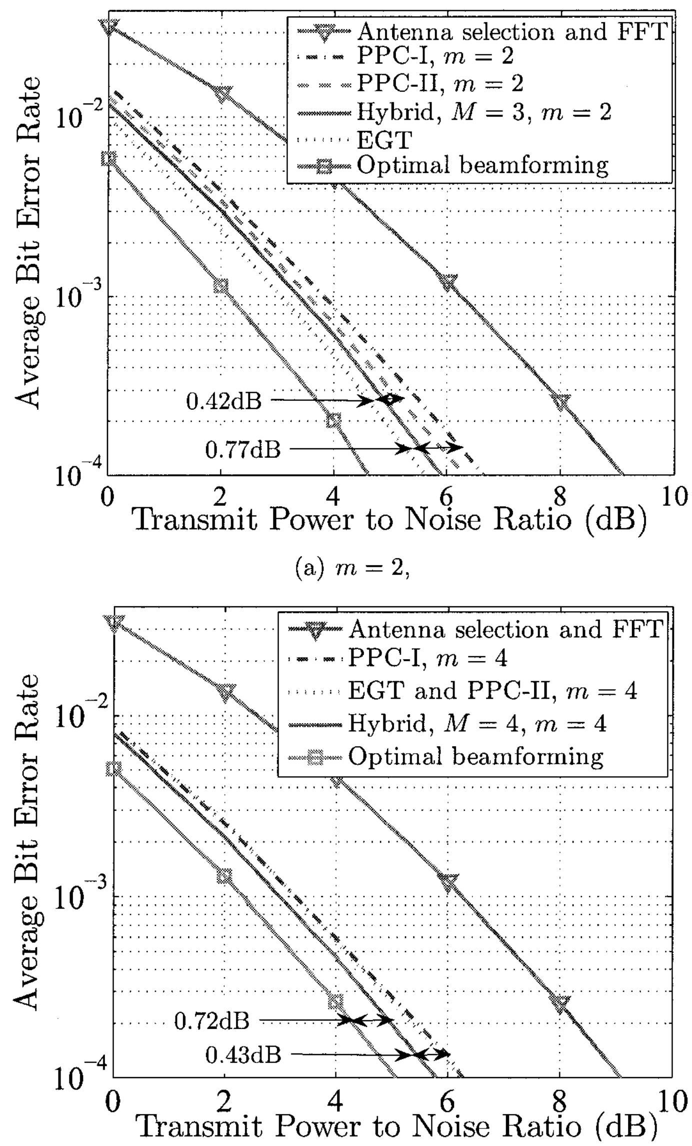


Figure 2.9: Required Transmit-power-to-Noise-Ratio (TNR) versus number of feedback bits used in order to achieve an average BER of 10^{-3}.

This implies that for small numbers of feedback bits, the PPC-II method with the almost-optimal procedure, significantly outperforms the PPC-I method. Figure 2.10 (b) shows when $m = 4$ they have almost the same average BER curves. As m increases, performances of the PPC methods become similar to each other and to the performance of the EGT method.

Another important observation is that as m increases, hybrid method significantly outperforms the EGT which requires infinite number of feedback bits. As Figure 2.10 (b) shows, at the average BER of 10^{-4}, hybrid method with four selected antennas has 0.41 dB gain in TNR over the EGT and only 0.77 dB TNR loss compared to the optimal beamforming. This implies that by using few bits of information about the magnitude of channel coefficients, we can avoid wasting energy on sub-channels with insignificant gains and improve the performance.

Figure 2.11, shows the average SNR curves of the discussed methods versus the



(a) $m = 2$,

(b) $m = 4$,

Figure 2.10: Average BER versus the transmit-power-to-noise-ratio (TNR), $\frac{P}{\sigma_n^2}$, in a 5×1 system, for (a) $m = 2$ and (b) $m = 4$.

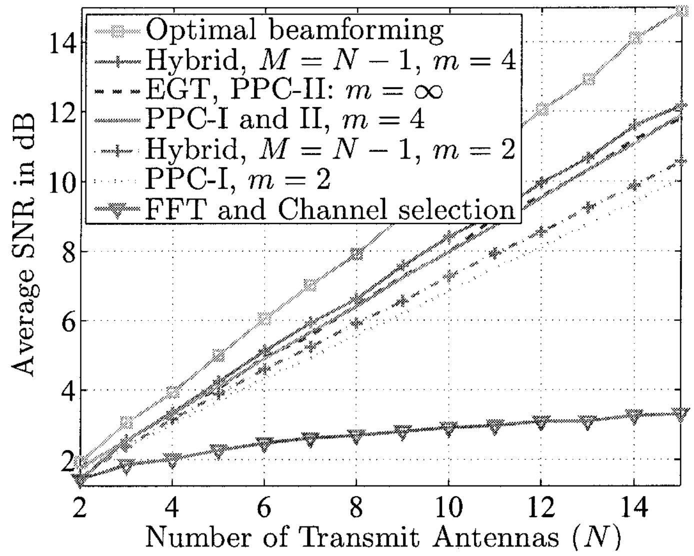
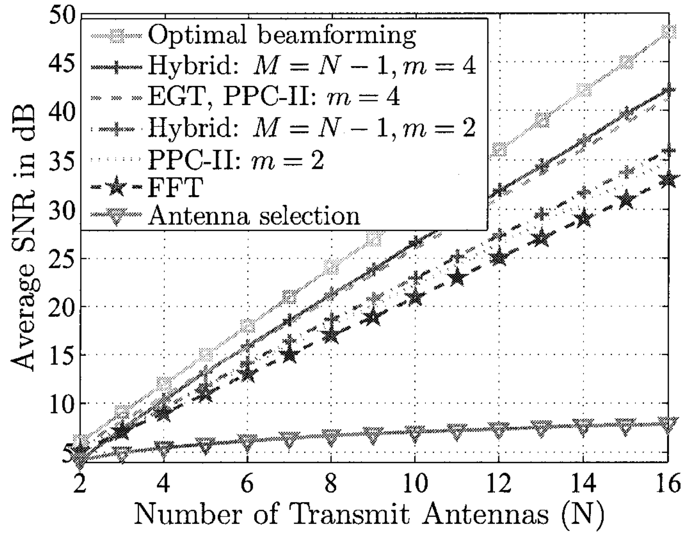
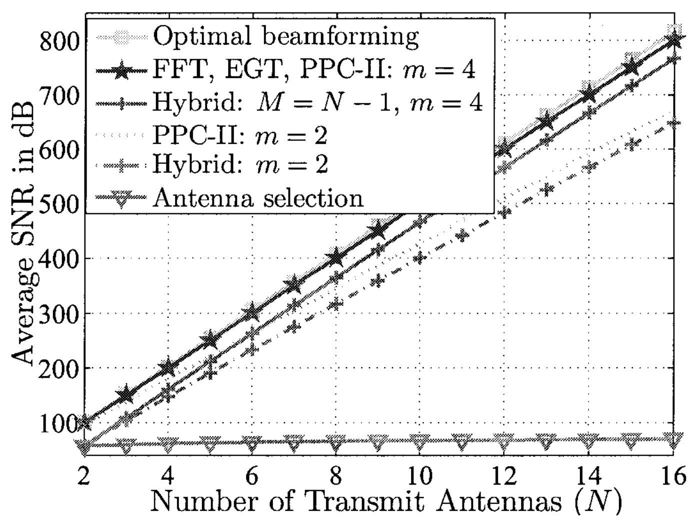


Figure 2.11: Average SNR curves the number of transmit antennas in an uncorrelated Rayleigh fading channel, with $\frac{P}{\sigma_n^2} = 1$.

number of transmit antennas N in Rayleigh fading channel, when the TNR is fixed at one. This figure verifies all our observations from Figure 2.10. In Figure 2.12, the average SNR of these methods are compared in Rician fading channels with Rician factors of $K = 2$ and $K = 50$, respectively, using a uniform linear array of omnidirectional antennas. Note that Figure 2.11 also represents a Rician channel with $K = 0$. In Rician channels and with uniform linear array of omnidirectional antennas, the FFT method significantly outperforms the channel selection method (e.g., in a 6×1 system the FFT method has 6.8dB gain in average SNR over the antenna selection method). From Figure 2.12, we observe that as the Rician factor increases (i.e., when the line-of-sight component is very strong) the performance of the FFT method tends toward the performance of the OBS and outperforms all the other methods by using the least bits of feedback (i.e., $\lceil \log_2 N \rceil$ bits). Figures 2.13 (a) and (b) show the average SNR performance of all the above methods in correlated Rayleigh fading



(a) $K = 2$



(b) $K = 50$

Figure 2.12: Average SNR curves versus the number of transmit antennas for $\frac{P}{\sigma_n^2} = 1$, in a Rician fading channel with Rician factor of (a) $K = 2$, (b) $K = 50$.

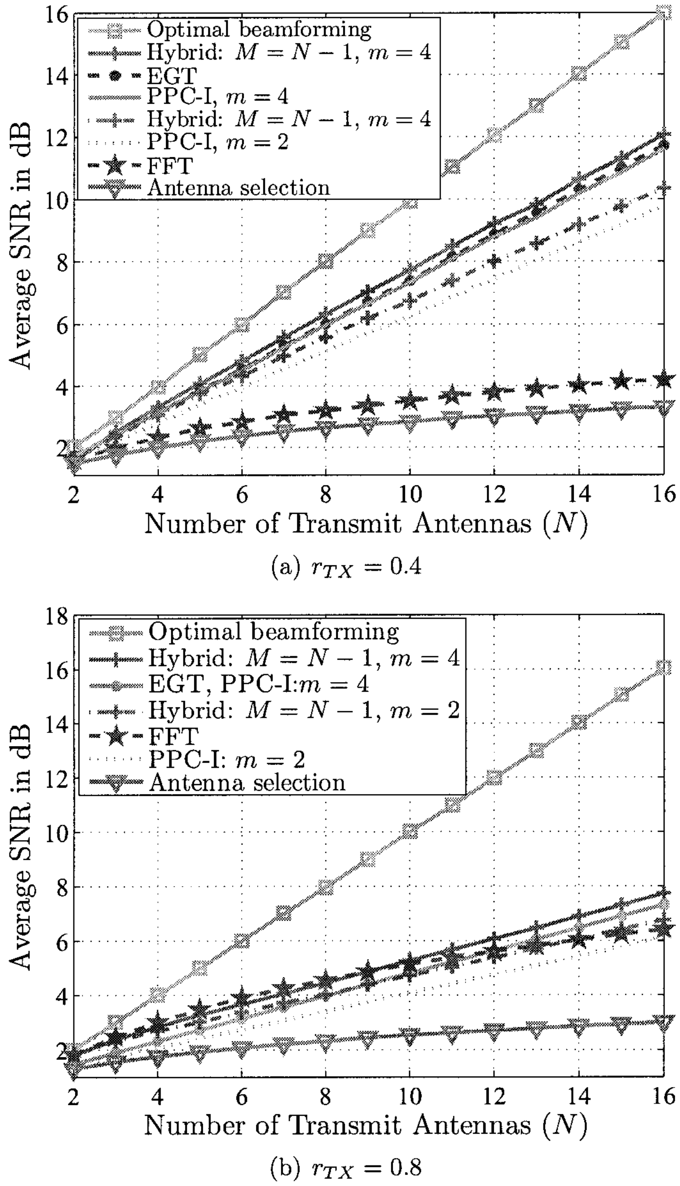


Figure 2.13: Average SNR curves versus the number of transmit antennas in a correlated Rayleigh fading channel, with (a) $r_{TX} = 0.4$, (b) $r_{TX} = 0.8$.

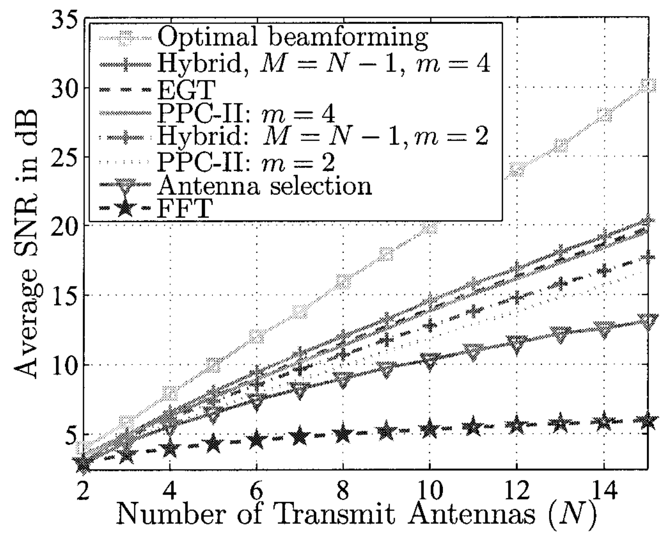


Figure 2.14: Average SNR curves versus number of transmit antennas using a uniform array of non-omnidirectional antennas, for $\frac{P}{\sigma_n^2} = 1$.

channels. We consider the single coefficient spatial correlation model introduced by Zelst and Hammerschmidt [40] with the coefficient $r_{TX} = 0.2$ and $r_{TX} = 0.8$. We observe that the FFT-based antenna selection outperforms the antenna selection in correlated fading channels. As the channel correlation increases the performance improvement of FFT method over antenna selection increases. In order to study the impact of non-omnidirectional antennas, in Figure 2.14 we consider a channel where h_i's are i.i.d. complex Gaussian random variables with unit variance and mean of $E[h_i] = \frac{\sin(\frac{N}{2}\theta - i\pi)}{\sqrt{N}\sin(\frac{1}{2}\theta - \frac{i\pi}{N})}$, where θ represents random direction of propagation which is uniformly distributed between $(-\pi, \pi]$. We observe that using this linear array of non-omnidirectional antennas, the antenna selection method outperforms the FFT method. From this figure we conclude that, by employing non-omnidirectional antennas, the antenna selection method may outperform the FFT method.

Figure 2.15 depicts the required Transmit Power to Noise Ratio (TNR), in order

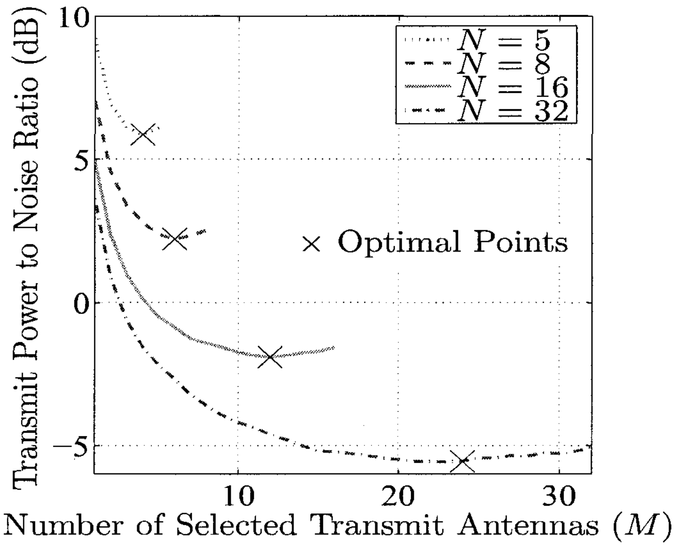


Figure 2.15: The required Transmit Power to Noise Ratio (TNR), in order to achieve an average BER of 10^{-4}, versus the number of selected transmit antennas M in hybrid method for various numbers of transmit antennas N in Rayleigh fading channel.

to achieve an average BER of 10^{-4}, versus the number of selected transmit antennas M in hybrid method for various numbers of transmit antennas N in Rayleigh fading channel. The number of bits allocated for phase adjustment is considered to be large ($m \geq 4$ bits). We observe for various numbers of transmit antennas N that in Rayleigh fading channel, the optimal performance is achieved when the number of selected transmit antennas M is about $\frac{3}{4}N$. This means that if we do not transmit signals over $\frac{1}{4}N$ antenna elements which have smaller gains we can save energy and as a result achieve better performance compared to PPC and EGT methods.

2.5 Summary and Conclusions

In this chapter we studied several transmit diversity schemes for a MISO wireless system with limited feedback and evaluated and compared their BER and SNR performances. Aiming to evaluate the efficiency and importance of different components of CSI, we studied the two extreme schemes, the antenna selection (using only magnitude information) and the partial phase combining (PPC) (using only phase information) and we proposed a hybrid method which applies both the magnitude and phase information. For $N > 2$, our proposed hybrid method applies phase adjustment on signals of stronger subchannels and can significantly outperform the PPC and even the EGT methods by avoiding the energy waste over the insignificant subchannels. We observed that by applying few bits on magnitude information, we can attain better performance compared with the PPC method by using hybrid method with the same total number of bits. This method also requires considerably reduced electronic circuitry and less computational complexity at the transmitter compared to the PPC and EGT methods as it performs PPC only on selected subchannels. Our results reveal that for a Rayleigh channel, selection of at most 75% of antenna elements results in best performance when beamformer only adjusts the phase of transmit signals. We suggest to use only about half of transmit antennas in order to achieve a close to optimal performance and a reduced price of the transmitter complexity. For 2-transmit antenna systems, our practical proposed hybrid method applies one-bit magnitude information to adjust the power of the phase steered transmitted signals and performs as close as 0.14 dB to the OBS. This implies that applying only one bit magnitude information is enough to elevate the performance of PPC method very close to the optimal beamforming. The proposed hybrid scheme is simple to implement and practical and allows better use of resources (the feedback bits) compared to the PPC and antenna selection methods. We also proposed an almost-optimal procedure for finding optimal PPC codewords which reduces the computational complexity from the order of $O\left(2^{m(N-1)}\right)$ to $O\left(N-1\right)$. This scheme significantly outperforms the PPC-I method for small amount of feedback bits and as the number of feedback bits increases performs the same as the EGT method by having much less computational complexity and using far fewer feedback bits. By applying transmit antenna selection on the FFT of the channel matrix (FFT method) we substantially improved the performance of the antenna selection method in systems with correlated fading channels and also systems employing uniform linear array of omnidirectional antennas. If the employed antennas are non-omnidirectional, the antenna selection method may outperform the FFT.

Chapter 3

Performance Analysis under Channel Estimation Errors and Feedback Delay

3.1 Introduction

In the previous chapter, the channel information provided to the transmitter is assumed to be noiseless and error-free. This assumption may not be true in practice due to channel errors. The channel errors can be characterized as follows,

1. The errors due to the estimation of the channel at the receiver: these errors impact the performance by degrading the optimality and accuracy of the detection at the receiver and optimality of beamformer at the transmitter. These errors arise due to, for instance, time or frequency separation between the pilot and the signal [41, 42], or inaccurate signal estimation [42, 43].

2. The errors due to the delay involved in the feedback link: as a result of the feedback delay, the feedback information becomes outdated which results in performance degradations by making the transmitter less efficient [11,12]. These errors arise due to, for instance, high mobile velocities [44] or fast fading channel environments. [12]

3. Quantization errors in the feedback link: these errors are due to the limited capacity of the feedback channel that arises the need for quantization of the channel parameters. These errors have already been considered in the performance of limited-feedback transmit diversity schemes in previous chapter.

Since all closed-loop transmit diversity schemes require some knowledge of the channel coefficients which have to be estimated at the receiver and conveyed to the transmitter through a feedback channel, channel errors inevitably affect the performance of the closed-loop transmit diversity schemes. Thus, from a practical and theoretical point of view, it is important to quantify the performance degradation of the practical limited-feedback transmit diversity schemes under channel errors such as feedback delay and channel estimation errors. In this chapter, we study the impact of both the channel estimation errors and feedback delay on the performance of several practical limited-feedback transmit diversity schemes.

Few published analytical results exist for the effect of the channel errors on the performance of transmit diversity schemes and most of them consider the impact of only one of the possible channel errors. The average BER performance of closed-loop and open-loop transmit diversity schemes are analyzed and compared in the presence of feedback delays by Onggosanusi *et al.* in [11]. The average BER expression for the OBS in the presence of feedback delay is derived by Choi in [12]. The downlink capacity with delayed feedback of channel state information has been derived by Huang *et al.* [13]. The closed-form average BER expression for Alamouti scheme with imperfect channel estimation has been derived in [45]. The moment generating function (MGF) of output SNR in OBS with channel estimation errors is derived for MIMO systems by Chen and Tellambura in [46] and before that for MISO systems by Vanganuru and Annamalai [47]. In this thesis we analyze the joint and separate impact of channel estimation errors and feedback delays on the performance of transmit antenna selection, PPC and hybrid schemes.

The transmit antenna selection, hybrid, and PPC schemes are simple practical methods which can be easily deployed in wireless systems. It is thus important to quantify their performances degradation under practical conditions. For this reason we study and analyze the performance of these methods under channel estimation errors and feedback delay. We derive the pdf of the phase difference between the estimated and predicted channel coefficients and derive the average SNR of the PPC method in the presence of channel estimation errors and feedback delay. We also derive the pdf of the output SNR and the exact closed-form average SNR and BER expressions for transmit antenna selection under channel estimation errors and feedback delay in 2×1 systems. We show that the channel estimation errors and feedback delay have separable impacts on the SNR performance and the average SNR degradation due to channel estimation errors is the identical for these methods. We show that for all the transmit diversity schemes, channel estimation errors do not change the diversity gain compared to the perfect estimation case and degrade the performance by reducing the coding gain by an identical factor. We also showed that feedback delay results in reduction of diversity gain and if the channel estimation is perfect, feedback delay reduces the diversity gain to 1 (the same as the diversity gain of SISO) which leads to severe performance degradations.

As we observed in previous chapter, our proposed hybrid method is simple to implement, outperforms the PPC method and allows the better use of existing resources (the feedback bits) compared to the antenna selection and PPC methods. For example, in 2×1 systems, the hybrid method outperforms the PPC method with 0.50dB higher SNR using the same number of feedback bits. In order to quantify the impact of channel errors on this method, we analyze the performance of the hybrid method in the presence of channel estimation errors and feedback delay for 2×1 systems in two cases. In the first case, the correlation coefficient between the predicted and the estimated channel coefficients is known at the transmitter and the hybrid weighting coefficients are optimized using this knowledge (the aware hybrid scheme). In the second case, the correlation coefficient is not available at the transmitter and the hybrid weighting coefficients are not adapted to the channel errors (the unaware hybrid scheme). We derive the average SNR in both of these cases and observe that they perform very closely and adapting the weighting coefficients with the feedback delay does not improve the performance of hybrid method noticeably. We also observe that the hybrid method outperforms the PPC and antenna selection methods under channel errors and performs very closely to the OBS using just a few bits of feedback. We observe that the antenna selection and hybrid methods (using magnitude information) perform better than the PPC method (using only phase information) in the presence of feedback delay employing the same number of feedback bits. This implies superiority of employing magnitude information over employing phase information in case of experiencing feedback delay.

The rest of this chapter is organized as follows. Section 3.2 describes the system model. The performance analysis and derivation of average SNR of the PPC method under channel estimation errors and feedback delay are performed in Section 3.3. The performance analysis of the hybrid method under channel estimation errors and feedback delay with and without the knowledge of the channel errors at the transmitter is presented in Section 3.4 and average SNR is derived in both cases. The pdf of the output SNR and the exact closed-form expressions for average SNR and average BER of the transmit antenna selection with channel estimation errors and feedback delay are derived in Section 3.5 for 2×1 systems. The performance comparisons and evaluation results are provided in Section 3.6 and this is followed by the summary and conclusions in Section 3.7.

3.2 System Model

We consider a closed-loop transmit diversity system with N transmit and one receive antennas. We assume an i.i.d. Rayleigh flat-fading channel, additive white Gaussian noise with variance σ_n^2 and transmission power P. The channel response is assumed to be estimated at the receiver and predicted then quantized and sent back to the transmitter through a feedback channel with delay. The channel is estimated either by estimating the signal on each subchannel or by detecting a pilot sequence that was sent along with the signal.

The system model for general closed-loop transmit diversity systems is illustrated schematically in Figure 3.1. We denote the actual channel with $\mathbf{h}$, the estimated channel with $\mathbf{h}^e$ and the predicted channel with $\mathbf{h}^p$, where the superscripts e and p

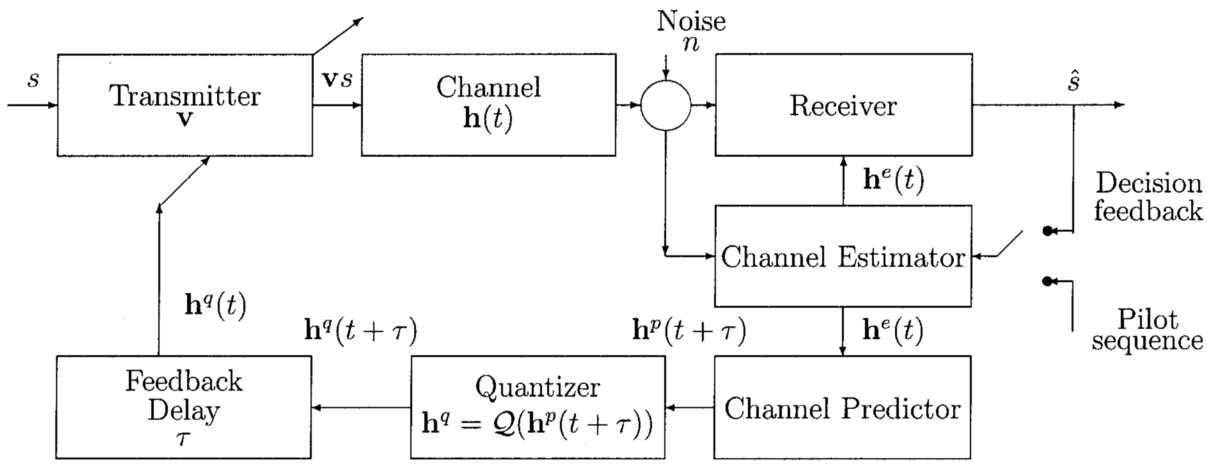


Figure 3.1: Block diagram of closed-loop transmit diversity schemes.

stand for estimated and predicted respectively. The channel estimator estimates the channel at time t, and the estimated channel vector, $\mathbf{h}^e(t)$, is used at the receiver for detection purposes and also at the channel predictor for predicting the channel at time $t + \tau$. This prediction is applied to compensate for the effect of delay of τ in the system. The predicted channel vector, $\mathbf{h}^p(t + \tau)$, is quantized to extract the feedback bits and the quantization output, $\mathcal{Q}(\mathbf{h}^p(t+\tau))$, is sent back to the transmitter through a feedback channel with delay τ. Thus the quantized value provided to the transmitter is $\mathcal{Q}(\mathbf{h}^p(t))$. Using this delayed quantized value of predicted channel, the transmitter applies a beamformer $\mathbf{v}$ on the transmit signal and transmits $\mathbf{v}s$ through the channel $\mathbf{h}(t)$.

To simplify our notations we drop the time index and denote the $\mathbf{h}(t)$, $\mathbf{h}^{\mathbf{e}}(t)$ and $\mathbf{h}^{\mathbf{p}}(t)$ by $\mathbf{h}$, $\mathbf{h}^{\mathbf{e}}$ and $\mathbf{h}^{\mathbf{p}}$ respectively. We assume that $\mathbf{h}$, $\mathbf{h}^{\mathbf{e}}$ and $\mathbf{h}^{\mathbf{p}}$ are zero mean circularly symmetric complex Gaussian (CSCG) random vectors with the same covariance matrix of $2\sigma^2\mathbf{I}_N$, where $\mathbf{I}_N$ is the $N \times N$ identity matrix. These random vectors are correlated as follows,

$$\mathbf{h} = \rho_e \mathbf{h}^e + \mathbf{e}^e, \tag{3.1}$$

$$\mathbf{h^e} = \rho_p \mathbf{h}^p + \mathbf{e}^p, \tag{3.2}$$

$$\mathbf{h} = \rho_e \rho_p \mathbf{h}^p + \rho_e \mathbf{e}^p + \mathbf{e}^e, \tag{3.3}$$

$$= \rho_d \mathbf{h}^p + \mathbf{e}^d. \tag{3.4}$$

We assume that $E[\mathbf{h}\mathbf{h}^{eH}] = 2\sigma^2 \rho_e \mathbf{I}_N$, $E[\mathbf{h}^e\mathbf{h}^{pH}] = 2\sigma^2 \rho_p \mathbf{I}_N$ and $E[\mathbf{h}\mathbf{h}^{pH}] = 2\sigma^2 \rho_e \rho_p \mathbf{I}_N$, and the correlation coefficients ρ_e and ρ_p are positive real numbers within $[0, 1]$. The assumption of having the same covariance matrices and positive real correlation coefficients does not reduce the generality of our model[1]. The $\mathbf{e}^e$, $\mathbf{e}^p$ and $\mathbf{e}^d$ are zero mean CSCG random vectors with covariance matrices of $\mathbf{R}_{\mathbf{e}^e} = 2\sigma^2(1-\rho_e^2)\mathbf{I}_N$, $\mathbf{R}_{\mathbf{e}^p} = 2\sigma^2(1-\rho_p^2)\mathbf{I}_N$ and $\mathbf{R}_{\mathbf{e}^d} = 2\sigma^2(\rho_e^2(1-\rho_p^2)+1-\rho_e^2)\mathbf{I}_N$ respectively. We assume that the random vectors $\mathbf{e}^e$, $\mathbf{e}^p$ and $\mathbf{h}^p$ are uncorrelated and as they have Gaussian distributions, they are independent as well.

3.3 Performance Analysis of PPC Method in the Presence of Feedback Delay and Channel Estimation Errors

In the PPC method with channel estimation errors and feedback delay, the channel information is estimated at the receiver and the phase combining coefficients are

[1]If two random vectors $\mathbf{Z}$ and $\mathbf{W}$ are jointly distributed with different covariance matrices and complex correlation coefficient, we can replace $\mathbf{Z}$ with $\tilde{\mathbf{Z}} \triangleq c\mathbf{Z}$ and define c such that the $\tilde{\mathbf{Z}}$ and $\mathbf{W}$ have the same covariances and the correlation coefficient between their components is a positive real number. Hence, we have: $E[|\tilde{Z}_i|^2] = E[|W_i|^2] = |c|^2 E[Z_i|^2]$, and $\mathrm{E}[\tilde{Z}_i W_i^*] = cE[Z_i W_i^*] \in \mathbb{R}^+$. Therefore the unique solution for c is given by, $c = \sqrt{\frac{\mathrm{E}[|W_i|^2]}{E[|Z_i|^2]}} \frac{E[Z_i{}^* W_i]}{|E[Z_i^* W_i]|}$.

extracted from the phase difference of predicted channel coefficients and sent to the transmitter through a feedback channel with delay. Therefore the transmitter applies the phase combining coefficients extracted from the predicted channel and the receiver uses the estimated channel which inevitably results in performance degradations. In this section we quantify this performance degradation by deriving the average SNR expression for PPC method in the presence of estimation error and feedback delay.

3.3.1 Performance Analysis for 2×1 Systems

The received signal of PPC scheme in 2×1 systems with channel estimation errors and feedback delay is,

$$r = \left(h_1 + h_2 e^{jQ(\angle h_1^p - \angle h_2^p)}\right) \frac{s}{\sqrt{2}} + n. \tag{3.5}$$

By using (3.1) the received signal becomes,

$$r = \frac{s}{\sqrt{2}} \rho_e \left(h_1^e + h_2^e e^{jQ(\angle h_1^p - \angle h_2^p)}\right) + \frac{s}{\sqrt{2}} \left(e_1^e + e_2^e e^{jQ(\angle h_1^p - \angle h_2^p)}\right) + n. \tag{3.6}$$

We consider the transmit signal power to be fixed, i.e. $|s|$ =constant, and by Lemma 2 in Appendix G the second term in (3.6) has Gaussian distribution with variance $2\sigma^2 P(1-\rho_e^2)$. As the receiver uses the estimated channel $\mathbf{h^e}$ for detection, the last two terms in (3.6) are considered as noise and the received SNR in this method is given by,

$$\begin{aligned}
\text{SNR} &= \frac{\frac{P\rho_e^2}{2}\left[|h_1^e + h_2^e e^{jQ(\angle h_1^p - \angle h_2^p)}|^2\right]}{2\sigma^2 P(1-\rho_e^2) + \sigma_n^2} & (3.7)\\
&= \frac{\frac{P\rho_e^2}{2}\left[|h_1^e|^2 + |h_2^e|^2 + h_1^e h_2^{e*} e^{-jQ(\angle h_1^p - \angle h_2^p)} + h_1^{e*} h_2^e e^{jQ(\angle h_1^p - \angle h_2^p)}\right]}{2\sigma^2 P(1-\rho_e^2) + \sigma_n^2} & (3.8)\\
&= \frac{\frac{P\rho_e^2}{2}\left[|h_1^e|^2 + |h_2^e|^2 + 2|h_1^e||h_2^e|\cos(\Delta_1 - \Delta_2 - U)\right]}{2\sigma^2 P(1-\rho_e^2) + \sigma_n^2}, & (3.9)
\end{aligned}$$

where $\Delta_1 = \angle h_1^e - \angle h_1^p$, $\Delta_2 = \angle h_2^e - \angle h_2^p$ and $U = (\angle h_1^p - \angle h_2^p) - Q(\angle h_1^p - \angle h_2^p)$. Knowing the fact that h_1^e and h_2^e have the same distributions and the magnitudes are independent of phases we can say that

$$\overline{\text{SNR}} = \frac{\frac{P\rho_e^2}{2}\left[2E|h_1^e|^2 + 2\left(E|h_1^e|\right)^2 E\left[\cos(\Delta_1 - \Delta_2 - U)\right]\right]}{2\sigma^2 P(1-\rho_e^2) + \sigma_n^2}. \quad (3.10)$$

Assuming that U, Δ_1 and Δ_2 are independent, Δ_1 and Δ_2 have the same distributions and U is uniformly distributed over $\left(-\frac{\pi}{2^m}, \frac{\pi}{2^m}\right)$ thus $E\left[\sin U\right] = 0$ and $E\left[\cos U\right] = \frac{2^m}{\pi}\sin(\frac{\pi}{2^m})$ we have,

$$\begin{aligned} E\left[\cos(\Delta_1 - \Delta_2 - U)\right] &= E\left[\cos(\Delta_1 - \Delta_2)\cos U + \sin(\Delta_1 - \Delta_2)\sin U\right] \quad (3.11) \\ &= \frac{2^m}{\pi}\sin(\frac{\pi}{2^m})\left((E\left[\cos\Delta_1\right])^2 + (E\left[\sin\Delta_1\right])^2\right). \quad (3.12) \end{aligned}$$

By replacing (3.12) into (3.10) we get,

$$\overline{\text{SNR}} = \frac{P\rho_e^2\left[E|h_1^e|^2 + \left(E|h_1^e|\right)^2 \frac{2^m}{\pi}\sin(\frac{\pi}{2^m})\left((E\left[\cos\Delta_1\right])^2 + (E\left[\sin\Delta_1\right])^2\right)\right]}{2\sigma^2 P(1-\rho_e^2) + \sigma_n^2}. \quad (3.13)$$

In order to derive the average SNR in (3.13) we have to find $E\left[\cos\Delta\right]$, $E\left[\sin\Delta\right]$ and the pdf of Δ. To find pdf of Δ we use the joint pdf of $|h^e|, |h^p|$ and Δ. Assuming that $h^e = h_R^e + ih_I^e$ and $h^p = h_R^p + ih_I^p$, the joint pdf of h_R^e, h_R^p, h_I^e and h_I^p is given by,

$$\begin{aligned} f_{h_R^e, h_R^p, h_I^e, h_I^p}(a_1, a_2, b_1, b_2) &= \frac{\exp\left(-\frac{1}{2}\left(a_1\ a_2\ b_1\ b_2\right)\mathbf{R}_h^{-1}\left(a_1\ a_2\ b_1\ b_2\right)^T\right)}{(2\pi)^2\sqrt{|\mathbf{R}_h|}} \quad (3.14) \\ &= \frac{\exp\left(-\frac{a_1^2+a_2^2+b_1^2+b_2^2-2\rho(a_1a_2+b_1b_2)}{2\sigma^2(1-\rho_p^2)}\right)}{(2\pi\sigma^2)^2(1-\rho_p^2)}, \quad (3.15) \end{aligned}$$

where $\mathbf{R}_h = \sigma^2\mathbf{I}_2 \otimes \begin{pmatrix} 1 & \rho_p \\ \rho_p & 1 \end{pmatrix}$ is the covariance matrix and $\otimes$ denotes the Kronecker product. We can easily find the joint pdf of $|h^e|, |h^p|, \angle h^e$ and $\angle h^p$ from (3.15) by converting rectangular coordinates to polar coordinates as

$$f_{|h^e|,|h^p|,\angle h^e,\angle h^p}(A^e, A^p, \theta^e, \theta^p) = \frac{A^e A^p \exp\left(-\frac{(A^e)^2+(A^p)^2-2A^eA^p\rho_p\cos(\theta^e-\theta^p)}{2\sigma^2(1-\rho_p^2)}\right)}{(2\pi\sigma^2)^2(1-\rho_p^2)}. \quad (3.16)$$

The joint pdf of $|h^e|, |h^p|$ and Δ, where $\Delta = \angle h^e - \angle h^p$, can be derived from the joint pdf of $|h^e|, |h^p|, \angle h^e, \angle h^p$ as

$$f_{|h^e|,|h^p|,\Delta}(A^e, A^p, \delta) = 2\pi f_{|h^e|,|h^p|,\angle h^e,\angle h^p}(A^e, A^p, \theta^e, \theta^p). \tag{3.17}$$

By taking integration from (3.17) with respect to A^e and A^p from 0 to ∞ and using the Taylor expansion $\exp(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}$ we find the pdf of Δ as follows,

$$\begin{aligned}
f_\Delta(\delta) &= \int_{A^e=0}^{\infty}\int_{A^p=0}^{\infty} \frac{A^e A^p \exp\left(-\frac{A^{e2}+A^{p2}}{2\sigma^2(1-\rho_p^2)}\right)\exp\left(\frac{A^e A^p \rho_p \cos(\delta)}{\sigma^2(1-\rho_p^2)}\right)}{2\pi\sigma^4(1-\rho_p^2)} dA^e dA^p & (3.18)\\
&= \int_{A^e=0}^{\infty}\int_{A^p=0}^{\infty} \frac{A^e A^p \exp\left(-\frac{A^{e2}+A^{p2}}{2\sigma^2(1-\rho_p^2)}\right)}{2\pi\sigma^4(1-\rho_p^2)} \sum_{k=0}^{\infty} \frac{\left(\frac{A^e A^p \rho_p}{\sigma^2(1-\rho_p^2)}\right)^k}{k!} \cos^k(\delta) dA^e dA^p & (3.19)\\
&= \sum_{k=0}^{\infty} \frac{1}{k!} a_k \cos^k(\delta), & (3.20)
\end{aligned}$$

where $a_k = \frac{\rho_p^k}{2\pi\sigma^{2k+4}(1-\rho_p^2)^{k+1}} \left(\int_{A^e=0}^{\infty} (A^e)^{k+1} \exp\left(-\frac{A^{e2}}{2\sigma^2(1-\rho_p^2)}\right) dA^e\right)^2$. Using the integral:

$$\int_{-\infty}^{\infty} \frac{|y|^n}{\sqrt{2\pi}} \exp(-\frac{y^2}{2}) dy = \begin{cases} 1 \times 3 \ldots \times (n-1), & n = 2l, \\ 2^{\frac{n-1}{2}} \left(\frac{n-1}{2}\right)! \sqrt{\frac{2}{\pi}}, & n = 2l+1, \end{cases} \tag{3.21}$$

we find the a_k as,

$$a_k = \frac{1}{2\pi} \rho_p^k (1-\rho_p^2) \begin{cases} 2^k \left[\left(\frac{k}{2}\right)!\right]^2, & \text{for } k = 2l, \\ \frac{\pi}{2} \frac{(k+1)!}{2^{\frac{k+1}{2}} \left(\frac{k+1}{2}\right)!}, & \text{for } k = 2l-1. \end{cases} \tag{3.22}$$

Having derived the pdf of $\Delta = \angle h - \angle h^p$ in (3.20) with a_k found in (3.22) we calculate $E[\cos\Delta]$ as follows,

$$E[\cos\Delta] = \sum_{k=0}^{\infty} \frac{1}{k!} a_k \int_{-\pi}^{\pi} \cos^{k+1}\delta d\delta. \tag{3.23}$$

The integrals of powers of cosine function in (3.23) have the closed forms of,

$$\begin{aligned}
&\int_{-\pi}^{\pi} \cos^{2n}\theta d\theta = \frac{(2n-1)!!}{(2n)!!} \left[\sin\theta \sum_{k=0}^{n-1} \frac{(2k)!!}{(2k+1)!!} \cos^{2k+1}\theta + \theta\right] \Big|_{-\pi}^{\pi} = \frac{(2n-1)!!}{(2n)!!}, \\
&\int_{-\pi}^{\pi} \cos^{2n+1}\theta d\theta = \frac{(2n)!!}{(2n+1)!!} \sin\theta \sum_{k=0}^{n} \frac{(2k)!!}{(2k+1)!!} \cos^{2k}\theta \Big|_{-\pi}^{\pi} = 0,
\end{aligned} \tag{3.24}$$

where double factorial of positive integer m is defined as,

$$m!! = \begin{cases} m(m-2)\dots 5\times 3\times 1, & \text{for } m \text{ odd},\\ m(m-2)\dots 6\times 4\times 2, & \text{for } m \text{ even},\\ 1, & \text{for } m=-1,0. \end{cases} \tag{3.25}$$

By applying (3.24) into (E.2) we get,

$$E\left[\cos\Delta\right] = \frac{\pi}{2}(1-\rho_p^2)\sum_{l=1}^{\infty}\frac{\rho_p^{2l-1}}{(2l-1)!}\left(\frac{(2l)!}{2^l l!}\right)^2\frac{(2l-1)!!}{(2l)!!}, \tag{3.26}$$

$$= \frac{\pi}{2}(1-\rho_p^2)\sum_{l=1}^{\infty}\rho_p^{2l-1}2l\left(\frac{(2l-1)!!}{(2l)!!}\right)^2, \tag{3.27}$$

$$= \frac{\pi}{4}\rho_p\left[1+\sum_{l=1}^{\infty}\left(\frac{(2l-1)!!}{(2l)!!}\right)^2\frac{\rho_p^{2l}}{l+1}\right]. \tag{3.28}$$

We also have that

$$E\left[\sin\Delta\right] = \int_{-\pi}^{\pi}\sum_{k=0}^{\infty}\frac{1}{k_1!}a_k\cos^k(\delta)\sin(\delta)d\delta = 0. \tag{3.29}$$

By applying (3.28) and (3.29) into (3.13) we derive the average SNR expression for PPC method in the presence on feedback delay and channel estimation errors as,

$$\overline{\text{SNR}} = \frac{P\rho_e^2\left[E|h_1^e|^2 + \left(E|h_1^e|\right)^2\pi 2^{m-4}\sin\frac{\pi}{2^m}\rho_p^2\left[1+\sum_{l=1}^{\infty}\left(\frac{(2l-1)!!}{(2l)!!}\right)^2\frac{\rho_p^{2l}}{l+1}\right]^2\right]}{2\sigma^2P(1-\rho_e^2)+\sigma_n^2} \tag{3.30}$$

$$= \frac{2\sigma^2P\rho_e^2\left[1+\rho_p^2\pi^2 2^{m-6}\sin\frac{\pi}{2^m}\left[1+\sum_{l=1}^{\infty}\left(\frac{(2l-1)!!}{(2l)!!}\right)^2\frac{\rho_p^{2l}}{l+1}\right]^2\right]}{2\sigma^2P(1-\rho_e^2)+\sigma_n^2}, \tag{3.31}$$

where (3.31) is derived knowing that $E|h_1^e| = \sqrt{\frac{\pi}{2}}\sigma$. If there is no feedback delay and no channel estimation errors, i.e. $\rho_p = 1$ and $\rho_e = 1$, and $2\sigma^2 = 1$ this result completely agrees with the average SNR of PPC derived in (2.22).

We observe from the expression derived for average SNR in (3.31) that the channel estimation errors and feedback errors affect the performance separately. Using the derived expression in (3.31) we can find the average SNR of the PPC method in two limiting cases of $\rho_e = 1$ (having only feedback delay error), and $\rho_p = 1$ (having only estimation error). The average SNR of the PPC method in the presence of feedback delay is derived by replacing $\rho_e = 1$ in (3.31) as,

$$\overline{\text{SNR}} = \frac{2\sigma^2 P}{\sigma_n^2}\left[1 + \rho_p^2 \pi^2 2^{m-6} \sin\frac{\pi}{2^m}\left[1 + \sum_{l=1}^{\infty}\left(\frac{(2l-1)!!}{(2l)!!}\right)^2 \frac{\rho_p^{2l}}{l+1}\right]^2\right]. \quad (3.32)$$

The average SNR of PPC method in the presence of channel estimation errors is derived by replacing $\rho_p = 1$ into (3.31) as,

$$\overline{\text{SNR}} = \frac{2\sigma^2 P \rho_e^2}{2\sigma^2 P(1-\rho_e^2) + \sigma_n^2}\left[1 + 2^{m-2}\sin\frac{\pi}{2^m}\right]. \quad (3.33)$$

We observe that the channel estimation errors result in SNR degradation of

$$\overline{\text{SNR}}_d = 10\log_{10}\left(\frac{\frac{2\sigma^2 P}{\sigma_n^2}(1-\rho_e^2)+1}{\rho_e^2}\right)\text{dB}, \quad (3.34)$$

compared to the case that the channel estimation is perfect ($\rho_e = 1$).

3.3.2 Performance Analysis for General MISO Systems

The received signal of PPC method in $N \times 1$ systems with feedback delay and estimation errors is given by,

$$r = \left(h_1 + h_2 e^{jQ(\angle h_1^p - \angle h_2^p)} + \ldots + h_N e^{jQ(\angle h_1^p - \angle h^p{}_N)}\right)\frac{s}{\sqrt{N}} + n \quad (3.35)$$

$$= \frac{s}{\sqrt{N}}\sum_{i=1}^{N} \rho_e h^e{}_i e^{jQ(\angle h_1^p - \angle h_i^p)} + \frac{s}{\sqrt{N}}\sum_{i=1}^{N} e_i^e e^{jQ(\angle h_1^p - \angle h_i^p)} + n. \quad (3.36)$$

By Lemma 2 in Appendix G, the second term in (3.36) has Gaussian distribution and the variance of total noise is $2\sigma^2 P(1-\rho_e^2) + \sigma_n^2$ and the received average SNR is

$$\overline{\text{SNR}} = \frac{P\rho_e^2}{N(2\sigma^2 P(1-\rho_e^2)+\sigma_n^2)} E\left[\left|\sum_{i=1}^{N} h_i^e e^{jQ(\angle h_1^p - \angle h_i^p)}\right|^2\right], \quad (3.37)$$

$$= \frac{P\rho_e^2 E\left[\sum_{p=1}^{N}\sum_{q=1}^{N} |h_p^e||h_q^e| e^{j\left(\angle h_p^e - \angle h_q^e + Q(\angle h_1^p - \angle h_p^p) - Q(\angle h_1^p - \angle h_q^p)\right)}\right]}{N(2\sigma^2 P(1-\rho_e^2)+\sigma_n^2)}. \quad (3.38)$$

By defining $\Delta_i = \angle h^e{}_i - \angle h_i^p$ and $U_i = (\angle h_1^p - \angle h_i^p) - Q(\angle h_1^p - \angle h_i^p)$ for $i = 1, \ldots, N$ we get,

$$\overline{\text{SNR}} = \frac{P\rho_e^2}{N(2\sigma^2 P(1-\rho_e^2)+\sigma_n^2)} E\left[\sum_{p=1}^{N}\sum_{q=1}^{N} |h_p^e||h_q^e| e^{j(\Delta_p-\Delta_q-U_p+U_q)}\right], \quad (3.39)$$

$$= \frac{P\rho_e^2}{N(2\sigma^2 P(1-\rho_e^2)+\sigma_n^2)} E\Big[\sum_{p=q=1}^{N} |h_p^e|^2 + \sum_{p=2}^{N} |h_p^e||h_1^e| e^{j(\Delta_p-\Delta_1-U_p)}$$
$$+\sum_{q=2}^{N} |h_1^e||h_q^e| e^{-j(\Delta_q-\Delta_1-U_q)} + \sum_{p\neq q=2}^{N}\sum_{q=2}^{N} |h_p^e||h_q^e| e^{j(\Delta_p-\Delta_q-U_p+U_q)}\Big]. (3.40)$$

Using the fact that $|h_i^e|$'s, Δ_i's and U_i's are independent and each have identical distribution and also that $E[\sin U] = 0$ and $E[\sin\Delta] = 0$ as shown in (3.29), we calculate the average SNR as follows,

$$\overline{\text{SNR}} = \frac{P\rho_e^2}{N(2\sigma^2 P(1-\rho_e^2)+\sigma_n^2)} \Big[NE\left[|h_1^e|^2\right] + 2(N-1)\left(E|h_1^e|\right)^2 \left(E[\cos\Delta]\right)^2 E[\cos U]$$
$$+(N-1)(N-2)\left(E|h_1^e|\right)^2 \left(E[\cos\Delta]\right)^2 \left(E[\cos U]\right)^2\Big], \quad (3.41)$$

$$= \frac{2\sigma^2 P\rho_e^2}{2\sigma^2 P(1-\rho_e^2)+\sigma_n^2}\left[1+\rho_p^2\left(\frac{N-1}{N}\right)2^{m-5}\pi^2\sin\left(\frac{\pi}{2^m}\right)\right.$$
$$\left.\times\left[1+(N-2)\frac{2^{m-1}}{\pi}\sin\left(\frac{\pi}{2^m}\right)\right]\left(1+\sum_{l=1}^{\infty}\left(\frac{(2l-1)!!}{(2l)!!}\right)^2\frac{\rho_p^{2l}}{l+1}\right)^2\right], \quad (3.42)$$

where (3.42) is derived by applying $E\left[\cos U\right] = \frac{2^m}{\pi}\sin(\frac{\pi}{2^m})$ and the expression derived for $E[\cos\Delta]$ in (3.28). In the case of no feedback delay and no channel estimation errors (i.e., $\rho_p = 1$ and $\rho_e = 1$) the average SNR in (3.42) converges to the average SNR for PPC derived in previous chapter (see (2.20)). We observe from (3.42) that the channel estimation errors and feedback delay (prediction errors) have independent impacts on the average SNR of PPC method.

The average SNR of PPC method in $N \times 1$ systems with feedback delay is derived by inserting $\rho_e = 1$ in (3.42) as,

$$\begin{aligned}\overline{\text{SNR}} &= \frac{2\sigma^2 P}{\sigma_n^2}\left[1+\rho_p^2\left(\frac{N-1}{N}\right)2^{m-5}\pi^2\sin\left(\frac{\pi}{2^m}\right)\left[1+(N-2)\frac{2^{m-1}}{\pi}\sin\left(\frac{\pi}{2^m}\right)\right]\right.\\ &\left.\times\left(1+\sum_{l=1}^{\infty}\left(\frac{(2l-1)!!}{(2l)!!}\right)^2\frac{\rho_p^{2l}}{l+1}\right)^2\right]. \qquad (3.43)\end{aligned}$$

The average SNR of PPC method in $N \times 1$ systems with estimation errors is derived by inserting $\rho_p = 1$ in (3.42) as,

$$\overline{\text{SNR}} = \frac{2\sigma^2 P\rho_e^2}{2\sigma^2 P(1-\rho_e^2)+\sigma_n^2}\left[1+\frac{N-1}{N}2^{m-1}\sin\frac{\pi}{2^m}+\frac{(N-1)(N-2)}{N}\frac{2^{2m-2}}{\pi}\sin^2\frac{\pi}{2^m}\right], \qquad (3.44)$$

Once again we observe that the channel estimation errors result in average SNR degradation of $\overline{\text{SNR}}_d = 10\log_{10}\left(\frac{\frac{2\sigma^2 P}{\sigma_n^2}(1-\rho_e^2)+1}{\rho_e^2}\right)$ dB compared to the perfect estimation case (see Section 2.3.4).

3.4 Performance Analysis of Hybrid Scheme in the Presence of Channel Estimation Errors and Feedback Delay

In our proposed hybrid method for 2-transmit antenna systems with channel estimation errors and feedback delay, the received signal is given by

$$\begin{aligned} r &= \left(g_1h_1 + g_2h_2e^{jQ(\angle h_1^p - \angle h_2^p)}\right)s + n \qquad (3.45)\\ &= \rho_e\left(g_1h_1^e + g_2h_2^e e^{jQ(\angle h_1^p-\angle h_2^p)}\right)s + \left(g_1e_1^e + g_2e_2^e e^{jQ(\angle h_1^p-\angle h_2^p)}\right)s + n, \qquad (3.46)\end{aligned}$$

where g_1 and g_2 are positive real numbers satisfying $g_1^2 + g_1^2 = 1$ to keep total transmission power fixed. In this method the average received SNR is given as,

$$\overline{\text{SNR}} = \frac{P\rho_e^2}{2\sigma^2 P(1-\rho_e^2)+\sigma_n^2} E\left[\left|g_1 h_1^e + g_2 h_2^e e^{jQ(\angle h_1^p - \angle h_2^p)}\right|^2 \Big| |h_1^p| > |h_2^p|\right]. \quad (3.47)$$

By defining $\Delta_1 = \angle h_1^e - \angle h_1^p$, $\Delta_2 = \angle h_2^e - \angle h_2^p$ and $U = (\angle h_1^p - \angle h_2^p) - Q(\angle h_1^p - \angle h_2^p)$ and considering that $|h_1^e||h_2^e|$ and $\cos(\Delta_1 - \Delta_2 - U)$ are uncorrelated given $|h_1^p| > |h_2^p|$ we get,

$$\begin{aligned}\overline{\text{SNR}} &= \frac{P\rho_e^2}{2\sigma^2 P(1-\rho_e^2)+\sigma_n^2}\Bigg(g_1^2 E\left[|h_1^e|^2 \Big| |h_1^p| > |h_2^p|\right] + g_2^2 E\left[|h_2^e|^2 \Big| |h_1^p| > |h_2^p|\right] \\ &\quad + 2g_1 g_2 E\left[|h_1^e||h_2^e| \Big| |h_1^p| > |h_2^p|\right] E\left[\cos(\Delta_1 - \Delta_2 - U)\right]\Bigg). \quad (3.48)\end{aligned}$$

Using the pdf of $X_1 = |h^e_{\arg_{i=1,2}\max |h_i^p|}|$ derived in (H.11) we find that,

$$E\left[|h_1^e|^2 \Big| |h_1^p| > |h_2^p|\right] = E[X_1^2] = 2\sigma^2\left(1 + \frac{\rho_p^2}{2}\right). \quad (3.49)$$

In order to find $E\left[|h_2^e|^2 \Big| |h_1^p| > |h_2^p|\right]$, we use the fact that the expression $|h_1^e|^2 + |h_2^e|^2$ is invariant to knowing $|h_1^p| > |h_2^p|$, thus $E\left[|h_1^e|^2 + |h_2^e|^2 \Big| |h_1^p| > |h_2^p|\right] = E\left[|h_1^e|^2 + |h_2^e|^2\right] = 4\sigma^2$. Therefore we have that

$$\begin{aligned}E\left[|h_2^e|^2 \Big| |h_1^p| > |h_2^p|\right] &= 4\sigma^2 - E\left[|h_1^e|^2 \Big| |h_1^p| > |h_2^p|\right], \\ &= 2\sigma^2\left(1 - \frac{\rho_p^2}{2}\right). \quad (3.50)\end{aligned}$$

By the fact that the expression $|h_1^e||h_2^e|$ is invariant to knowing $|h_1^p| > |h_2^p|$, we have $E\left[|h_1^e||h_2^e| \Big| |h_1^p| > |h_2^p|\right] = E\left[|h_1^e||h_2^e|\right] = \left[E|h_1^e|\right]^2 = \frac{\pi}{2}\sigma^2$. We also showed in Section 3.3 that $E\left[\cos(\Delta_1 - \Delta_2 - U)\right] = E\left[\cos(U)\right]\left(E\left[\cos(\Delta)\right]\right)^2 = \frac{2^m}{\pi}\sin(\frac{\pi}{2^m})\left(E\left[\cos(\Delta)\right]\right)^2$ and $E\left[\cos(\Delta)\right]$ is derived in (3.28). Using the derived expected values above and the derived expression for $E\left[\cos(\Delta)\right]$ in (3.28), we derive the average SNR as,

$$\overline{\text{SNR}} = \frac{2\sigma^2 P\rho_e^2}{2\sigma^2 P(1-\rho_e^2)+\sigma_n^2}\left[g_1^2\left(1 + \frac{\rho_p^2}{2}\right) + g_2^2\left(1 - \frac{\rho_p^2}{2}\right) + g_1 g_2 2^{m-1}\sin(\frac{\pi}{2^m}) E\left[\cos(\Delta)\right]^2\right]. \quad (3.51)$$

We consider two different cases for the hybrid method with channel estimation errors and feedback delay: the unaware case, where the correlation coefficient ρ_p is not available at the transmitter, and the aware case where the correlation coefficient ρ_p is available at the transmitter. In the unaware case we consider the hybrid method is not adapted to the channel errors, and the hybrid weighting coefficients, g_1 and g_2, are the same as derived in (2.28) and (2.29). By replacing g_1 and g_2 from (2.28) and (2.29) into (3.51) the average SNR in this case is derived as

$$\overline{\text{SNR}} = \frac{2\sigma^2 P\rho_e^2}{2\sigma^2 P(1-\rho_e^2)+\sigma_n^2}\left[1+\frac{\rho_p^2}{2}\frac{1+2^{2m-6}\sin^2\left(\frac{\pi}{2^m}\right)\pi^2\left(1+\sum_{l=1}^{\infty}\left(\frac{(2l-1)!!}{(2l)!!}\right)^2\frac{\rho_p^{2l}}{l+1}\right)^2}{\sqrt{1+2^{2m-2}\sin^2\left(\frac{\pi}{2^m}\right)}}\right]. \tag{3.52}$$

We observe that for $\rho_p = 1$ and $\rho_e = 1$ the above expression results in the same expression derived for the average SNR of hybrid method in Section (2.3.5). In this case we again observe that the average SNR degradation due to the channel estimation errors is the same as (3.34).

In the aware case, the real positive hybrid coefficients g_1 and g_2 are adopted with the feedback delay and are optimized such that the average SNR is maximized given which delayed channel coefficient has the largest magnitude. The derivation of g_1 and g_2 is as follows,

$$g_1, g_2 = \arg\max_{g_1,\ g_2; g_1^2+g_2^2=1} E\left[\left|g_1h_1+g_2h_2e^{jQ(\angle h_1^p-\angle h_2^p)}\right|^2 \left||h_1^p|>|h_2^p|\right.\right]. \tag{3.53}$$

Using the general expression for average SNR derived in (3.51)we have,

$$g_1, g_2 = \max_{g_1^2+g_2^2=1}\left[g_1^2\left(1+\frac{\rho_p^2}{2}\right)+g_2^2\left(1-\frac{\rho_p^2}{2}\right)+g_1g_2 2^{m-1}\sin\left(\frac{\pi}{2^m}\right)E\left[\cos(\Delta)\right]^2\right]. \tag{3.54}$$

We can calculate g_1 and g_2 by applying Lagrangian multipliers as follows,

$$\mathcal{L} = g_1^2\left(1+\frac{\rho_p^2}{2}\right) + g_2^2\left(1-\frac{\rho_p^2}{2}\right) + g_1 g_2 2^{m-1}\sin(\frac{\pi}{2^m})E\left[\cos(\Delta)\right]^2 + \lambda(g_1^2+g_2^2-1).$$

$$\frac{\partial\mathcal{L}}{\partial g_1} = 0 \Rightarrow\ g_1\left(1+\frac{\rho_p^2}{2}\right) + g_2 2^{m-2}\sin(\frac{\pi}{2^m})E\left[\cos(\Delta)\right]^2 + \lambda g_1 = 0. \tag{3.55}$$

$$\frac{\partial\mathcal{L}}{\partial g_2} = 0 \Rightarrow\ g_2\left(1-\frac{\rho_p^2}{2}\right) + g_1 2^{m-2}\sin(\frac{\pi}{2^m})E\left[\cos(\Delta)\right]^2 + \lambda g_2 = 0. \tag{3.56}$$

By canceling λ from (3.55) and (3.56), applying the constraint (i.e., $g_1^2+g_2^2=1$), doing some manipulations and applying the expression derived for $E\left[\cos(\Delta)\right]$ in (3.28), we find g_1 and g_2 as,

$$g_1 = \frac{1}{\sqrt{2}}\sqrt{1+\frac{1}{\sqrt{1+2^{2m-10}\sin^2\left(\frac{\pi}{2^m}\right)\pi^4\left(1+\sum_{l=1}^{\infty}\left(\frac{(2l-1)!!}{(2l)!!}\right)^2\frac{\rho_p^{2l}}{l+1}\right)^4}}}, \tag{3.57}$$

$$g_2 = \frac{1}{\sqrt{2}}\sqrt{1-\frac{1}{\sqrt{1+2^{2m-10}\sin^2\left(\frac{\pi}{2^m}\right)\pi^4\left(1+\sum_{l=1}^{\infty}\left(\frac{(2l-1)!!}{(2l)!!}\right)^2\frac{\rho_p^{2l}}{l+1}\right)^4}}}. \tag{3.58}$$

We observe that the optimized g_1 and g_2 depend on the prediction correlation coefficient ρ_p. In the case of no delay ($\rho_p = 1$), the g_1 and g_2 in (3.57) and (3.58) have the same expressions as g_1 and g_2 derived in Section (2.3.5). As we observe from the expressions of g_1 and g_2 in (3.57) and (3.58) for the aware hybrid method and Figure 3.2, the g_1 and g_2 do not change severely with ρ_p in the aware hybrid method and as m increases their difference becomes less.

By applying the adapted g_1 and g_2 in (3.57) and (3.58) into (3.51) we derive the average SNR in the aware hybrid method as,

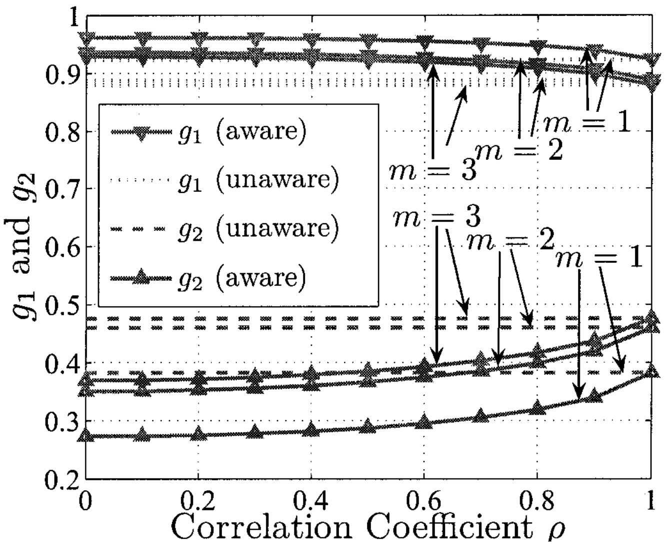


Figure 3.2: The hybrid weighting coefficients, g_1 and g_2, as functions of ρ_p for different values of m.

$$\overline{\text{SNR}} = \frac{2\sigma^2 P\rho_e^2 \left[1 + \frac{\rho_p^2}{2}\sqrt{1 + 2^{2m-10}\sin^2\left(\frac{\pi}{2^m}\right)\pi^4 \left(1 + \sum_{l=1}^{\infty}\left(\frac{(2l-1)!!}{(2l)!!}\right)^2 \frac{\rho_p^{2l}}{l+1}\right)^4}\right]}{2\sigma^2 P(1-\rho_e^2) + \sigma_n^2}. \tag{3.59}$$

If $\rho_p = 1$ and $\rho_e = 1$, the expression in (3.59) reduces to the average SNR derived in previous chapter for this method in (2.31). Once again, we observe that the feedback delay and channel estimation errors have disjoint impacts on the SNR performance and the average SNR degradation due to the channel estimation errors is the same as (3.34). The average SNR of aware hybrid method in the presence of feedback delay is derived from (3.59) as,

$$\overline{\text{SNR}} = \frac{2\sigma^2 P}{\sigma_n^2}\left[1 + \frac{\rho_p^2}{2}\sqrt{1 + 2^{2m-10}\sin^2\left(\frac{\pi}{2^m}\right)\pi^4\left(1 + \sum_{l=1}^{\infty}\left(\frac{(2l-1)!!}{(2l)!!}\right)^2 \frac{\rho_p^{2l}}{l+1}\right)^4}\,\right]. \tag{3.60}$$

The average SNR of aware hybrid method in the presence of estimation errors is derived from (3.59) as,

$$\overline{\text{SNR}} = \frac{2\sigma^2 P \rho_e^2}{2\sigma^2 P(1-\rho_e^2) + \sigma_n^2}\left[1 + \frac{1}{2}\sqrt{1 + 2^{2m-2}\sin^2\frac{\pi}{2^m}}\right] \tag{3.61}$$

By comparing the average SNR of the aware and unaware hybrid method in (3.59) and (3.52) and Figure 3.5, we observe that the aware hybrid method performs very closely to the unaware hybrid method and the performance improvement due to adapting the hybrid method to the feedback delay is marginal.

3.5 Performance Analysis of Transmit Antenna Selection under Channel Estimation Errors and Feedback Delay

In the antenna selection method the receiver finds the subchannel with the largest magnitude and informs the transmitter through a feedback channel and the transmitter subsequently transmits the signal from the selected antenna. In the presence of channel estimation errors and feedback delay, the information provided to the transmitter is extracted from the predicted channel and sent through a feedback channel with delay. Thus the selected subchannel at the transmitter may not be the actual strongest subchannel. The receiver applies the imperfect estimated channel for detection and all this leads to performance degradation. In this section we quantify this performance degradation by deriving the exact closed-form expressions for average SNR and BER of transmit antenna selection under channel estimation errors and feedback delay in 2-transmit antenna systems.

The received signal of transmit antenna selection in 2×1 systems with channel estimation errors and feedback delay is

$$
\begin{aligned}
r &= h_{\arg\max_{i=1,2}\{|h_i^p|\}} s + n &(3.62)\\
&= \rho_e {h^e}_{\arg\max_{i=1,2}\{|h_i^p|\}} s + {e^e}_{\arg\max_{i=1,2}\{|h_i^p|\}} s + n. &(3.63)
\end{aligned}
$$

By the fact that e_i^e's are independent of h_i^p's, the variance of the equivalent noise in this system is $2\sigma^2 P(1-\rho_e^2) + \sigma_n^2$ and the received SNR is given by,

$$
\mathrm{SNR} = \frac{P\rho_e^2}{2\sigma^2 P(1-\rho_e^2) + \sigma_n^2} |{h^e}_{\arg\max_{i=1,2}\{|h_i^p|\}}|^2. \quad (3.64)
$$

In order to derive the average SNR and BER we need to have the joint distribution of $|h^e|$ and $|h^p|$. By taking integral of $f_{|h^e|,|h^p|,\Delta}(A^e, A^p, \delta)$ in (3.17) with respect to Δ we derive the joint pdf of $|h^e|$ and $|h^p|$ as follows,

$$
\begin{aligned}
f_{|h^e|,|h^p|}(A^e, A^p) &= \int_{-\pi}^{\pi} f_{|h^e|,|h^p|,\Delta}(A^e, A^p, \delta) d\delta\\
&= \frac{2\pi A^e A^p}{(2\pi\sigma^2)^2(1-\rho_p^2)} \exp\left(-\frac{A^{e2} + A^{p2}}{2\sigma^2(1-\rho_p^2)}\right) \int_{-\pi}^{\pi} \exp\left(\frac{A^e A^p \rho_p \cos(\delta)}{\sigma^2(1-\rho_p^2)}\right) d\delta\\
&= \frac{A^e A^p}{\sigma^4(1-\rho_p^2)} \exp\left(-\frac{A^{e2} + A^{p2}}{2\sigma^2(1-\rho_p^2)}\right) I_0\left(\frac{A^e A^p \rho_p}{\sigma^2(1-\rho_p^2)}\right), \quad (3.65)
\end{aligned}
$$

where the $I_0(x)$ is the zero-order first kind modified Bessel function defined as

$$
I_0(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp(x \cos\delta) d\delta.
$$

In order to derive the pdf of $Z \triangleq |{h^e}_{\arg\max_{i=1,2}\{|h_i^p|\}}|^2$ in (3.64), we first derive the joint pdf of the sorted $|h_i^p|$'s and their corresponding $|h_i^e|$'s. We define $(\check{\mathbf{X}}, \check{\mathbf{Y}}) \triangleq$

$[(|h_1^e|, |h_1^p|), ..., (|h_N^e|, |h_N^p|)]$ with the joint pdf of

$$f_{\check{\mathbf{X}},\check{\mathbf{Y}}}(\check{\mathbf{x}}, \check{\mathbf{y}}) = \prod_{i=1}^{N} f_{|h_i^e|,|h_i^p|}(\check{x}_i, \check{y}_i). \tag{3.66}$$

The joint pdf of $(\mathbf{X}, \mathbf{Y}) = [(X_1, Y_1), (X_2, Y_2), ..., (X_N, Y_N)]$, where the Y_i's are the sorted $\check{Y}_i$'s such that $Y_1 \geq Y_2 \geq ... \geq Y_N$ and X_i's are the corresponding $\check{X}_i$'s to the sorted $\check{Y}_i$'s, can be calculated as,

$$\begin{aligned} f_{\mathbf{X},\mathbf{Y}}(\mathbf{x}, \mathbf{y}) &= \sum_{\nu \in \mathcal{U}} f_{\check{\mathbf{X}},\check{\mathbf{Y}}}\left(\nu(\check{\mathbf{x}}, \check{\mathbf{y}})\right) \\ &= \sum_{\nu \in \mathcal{U}} \prod_{i=1}^{N} f_{|h_i^e|,|h_i^p|}(x_i, y_i), \end{aligned} \tag{3.67}$$

where $\mathcal{U}$ is the set of all permutations of $\{(\check{X}_1, \check{Y}_1), (\check{X}_2, \check{Y}_2), ..., (\check{X}_N, \check{Y}_N)\}$ and $\nu(.)$ represents a permutation on $\{(\check{X}_1, \check{Y}_1), (\check{X}_2, \check{Y}_2), ..., (\check{X}_N, \check{Y}_N)\}$ such that $\check{Y}_1 \geq \check{Y}_2 \geq ... \geq \check{Y}_N$. By applying (3.65) into (3.67) we get,

$$\begin{aligned} f_{\mathbf{X},\mathbf{Y}}(\mathbf{x}, \mathbf{y}) &= \sum_{\nu \in \mathcal{U}} \prod_{i=1}^{N} \frac{1}{\sigma^4(1-\rho_p^2)} x_i y_i \exp\left(-\frac{x_i^2 + y_i^2}{2\sigma^2(1-\rho_p^2)}\right) I_0\left(\frac{x_i y_i \rho_p}{\sigma^2(1-\rho_p^2)}\right) \\ &= \frac{N!}{\left(\sigma^4(1-\rho_p^2)\right)^N} \exp\left(-\frac{\|\mathbf{x}\|^2 + \|\mathbf{y}\|^2}{2\sigma^2(1-\rho_p^2)}\right) \prod_{i=1}^{N} x_i y_i I_0\left(\frac{x_i y_i \rho_p}{\sigma^2(1-\rho_p^2)}\right) \end{aligned} \tag{3.68}$$

if $y_1 \geq y_2 \geq ... \geq y_N \geq 0$ and $x_i \geq 0$, $\forall i$, and zero elsewhere.

In a 2×1 system, the joint pdf in (3.68) is reduced to:

$$\begin{aligned} f_{X_1,X_2,Y_1,Y_2}(x_1, x_2, y_1, y_2) &= \frac{2x_1x_2y_1y_2}{\sigma^8(1-\rho_p^2)^2} \exp\left(-\frac{|x_1|^2 + |x_2|^2 + |y_1|^2 + |y_2|^2}{2\sigma^2(1-\rho_p^2)}\right) \\ &\quad \times I_0\left(\frac{x_1 y_1 \rho_p}{\sigma^2(1-\rho_p^2)}\right) I_0\left(\frac{x_2 y_2 \rho_p}{\sigma^2(1-\rho_p^2)}\right), \end{aligned} \tag{3.69}$$

for $y_1 \geq y_2 \geq 0$, $x_1 \geq 0$ and $x_2 \geq 0$, and is zero elsewhere. The pdf of $X_1 = |h^e{}_{\arg\max\{h_i^p\}}|$ can be derived by taking integral of $f_{X_1,X_2,Y_1,Y_2}(x_1, x_2, y_1, y_2)$ with respect to x_2, y_1 and y_2:

$$f_{X_1}(x_1) = \int_{x_2=0}^{\infty} \int_{y_1=0}^{\infty} \int_{y_2=0}^{y_1} f_{X_1,X_2,Y_1,Y_2}(x_1, x_2, y_1, y_2) dy_2 dy_1 dx_2. \tag{3.70}$$

By doing several integrations, manipulations and simplifications the exact closed-form expression for pdf of X_1 is derived in Appendix H as,

$$f_{X_1}(x_1) = \frac{2x_1}{\sigma^2}\left[\exp\left(-\frac{x_1^2}{2\sigma^2}\right) - \frac{\exp\left(-\frac{x_1^2}{2\sigma^2(1-\frac{\rho_p^2}{2})}\right)}{2(1-\frac{\rho_p^2}{2})}\right]u(x_1), \tag{3.71}$$

where $u(.)$ denotes the unit step function. Using the pdf of X_1 we can find the pdf of $Z = X_1^2$ in $\mathrm{SNR} = |\mathrm{h}_{\arg\max\{\mathrm{h}_i^{\mathrm{P}}\}}|^2\frac{\mathrm{P}}{\sigma_n^2} = \mathrm{Z}\frac{\mathrm{P}}{\sigma_n^2}$ as follows,

$$f_Z(z) = \frac{f_{X_1}(\sqrt{z})}{2\sqrt{z}} = \frac{1}{\sigma^2}\left[\exp\left(-\frac{z}{2\sigma^2}\right) - \frac{1}{2(1-\frac{\rho_p^2}{2})}\exp\left(-\frac{z}{2\sigma^2(1-\frac{\rho_p^2}{2})}\right)\right]u(z) \tag{3.72}$$

Using the pdf of Z, we calculate the average SNR of the antenna selection method in the presence of feedback delay and channel estimation errors as follows,

$$\overline{\mathrm{SNR}} = E\left[\mathrm{SNR}\right] = \frac{P\rho_e^2}{2\sigma^2 P(1-\rho_e^2)+\sigma_n^2}E[Z] = \frac{2\sigma^2 P\rho_e^2}{2\sigma^2 P(1-\rho_e^2)+\sigma_n^2}\left(1+\frac{\rho_p^2}{2}\right). \tag{3.73}$$

In the special case where $\rho_p = 1$ and $\rho_e = 1$, this result completely agrees with the average SNR of antenna selection method in the previous chapter. We again observe that the channel estimation errors and feedback delay (prediction errors) have independent impacts on the average SNR and the average SNR degradation to due channel estimation errors is the $\overline{\mathrm{SNR}_d}$ in (3.34).

In order to find the average BER using the Lemma 1 in Appendix D we need $F_Z(z)$. The cdf of random variable Z is given by,

$$F_Z(z) = \left(1 - 2\exp\left(-\frac{z}{2\sigma^2}\right) + \exp\left(-\frac{z}{2\sigma^2(1-\frac{\rho_p^2}{2})}\right)\right)u(z). \tag{3.74}$$

Observing that $F_Z(0) = 0$, we apply Lemma 1 in Appendix D to derive the average BER as follows,

$$P_e = \frac{\sqrt{\beta}}{2\sqrt{2\pi}}\int_0^\infty z^{-\frac{1}{2}}e^{-\frac{\beta z}{2}}F_Z(z)dz. \tag{3.75}$$

By using the definition of Gamma function $\Gamma(n+1) = b^{n+1}\int_0^\infty x^n e^{-bx}dx$ where $\Gamma(\frac{1}{2}) =$

$\sqrt{\pi}$, we derive the exact closed-form expression for average BER of transit antenna selection with BPSK and QPSK modulation in the presence of channel estimation errors and feedback delay as follows,

$$P_e = \frac{\sqrt{\beta}\Gamma(\frac{1}{2})}{2\sqrt{2\pi}} \left[\frac{1}{\sqrt{\frac{\beta}{2}}} + \frac{1}{\sqrt{\frac{\beta}{2} + \frac{1}{2\sigma^2(1-\frac{\rho_p^2}{2})}}} - \frac{2}{\sqrt{\frac{\beta}{2} + \frac{1}{2\sigma^2}}} \right], \tag{3.76}$$

$$= \frac{1}{2} \left[1 + \sqrt{\frac{\beta}{\beta + \frac{1}{\sigma^2(1-\frac{\rho_p^2}{2})}}} - 2\sqrt{\frac{\beta}{\beta + \frac{1}{\sigma^2}}} \right], \tag{3.77}$$

where $\beta = \frac{2P\rho_e^2}{(2\sigma^2 P(1-\rho_e^2)+\sigma_n^2)\log_2 D}$. If $\rho_p = 1$, $\rho_e = 1$ and $2\sigma^2 = 1$, the average BER derived in (3.77) completely agrees with the average BER found for this method in Section (2.3.3).

In the case of perfect channel estimation (i.e., $\rho_e = 1$), the average BER of antenna selection in the presence of feedback delay has the same expression as in (3.77) with $\beta = \frac{2P}{\sigma_n^2 \log_2 D}$. Using Appendix I, we find the diversity gain and coding gain of antenna selection method in the presence of feedback delay as follows,

$$G_d = \begin{cases} 1, & \text{if } \rho_p \neq 1, \\ 2, & \text{if } \rho_p = 1. \end{cases}, \quad G_c = \begin{cases} \frac{2-\rho_p^2}{1-\rho_p^2}, & \text{if } \rho_p \neq 1, \\ \sqrt{\frac{2}{3}}, & \text{if } \rho_p = 1. \end{cases} \tag{3.78}$$

We observe that in the presence of feedback delay, the diversity gain of the antenna selection method in 2×1 systems is reduced to 1 which is the same as the diversity gain in SISO systems.

If the channel estimation errors is not perfect (i.e., $\rho_e \neq 1$), error floors happen if ρ_e is fixed. However in most estimation schemes ρ_e increases with $\frac{P}{\sigma_n^2}$ and the variance of the estimation error decreases as the SNR of the data symbols increases. We assume that $\rho_e^2 = \frac{2\sigma^2\kappa P}{2\sigma^2\kappa P+\sigma_n^2}$ where κ depends on the length of the training sequence, the estimation algorithm and the rate of channel variations with time. We observe that in this case, the diversity gain remains the same as that of the perfect channel estimation case and the coding gain decreases by a factor of $\frac{\kappa}{\kappa+1}$. This is true for all transmit diversity schemes as in the presence of channel estimation errors with $\rho_e^2 = \frac{2\sigma^2\kappa P}{2\sigma^2\kappa P+\sigma_n^2}$ we have,

$$\beta = \frac{2P\rho_e^2}{(2\sigma^2 P(1-\rho_e^2)+\sigma_n^2)\log_2 D} = \frac{\kappa\sigma^2\beta_o^2}{\sigma^2\beta_o(1+\kappa)+\frac{1}{\log_2 D}}, \tag{3.79}$$

where $\beta_o = \frac{2P}{\sigma_n^2 \log_2 D}$ is the β in the perfect channel estimation case. Thus as $\beta \to \infty$, $\beta \to \left(\frac{\kappa}{\kappa+1}\right)\beta_o$ which by Appendix I implies that $G_d = G_d^o$ and $G_c = G_c^o \frac{\kappa}{\kappa+1}$. We conclude that for all the transmit diversity schemes, channel estimation errors do not change the diversity gain and degrade the performance by reducing the coding gain by a factor of $\frac{\kappa}{\kappa+1}$. We also showed that feedback delay results in reduction of diversity gain and if the channel estimation is perfect, feedback delay reduces the diversity gain to 1 (the same as the diversity gain of SISO).

If $\rho_e^2 = \frac{2\sigma^2\kappa P}{2\sigma^2\kappa P+\sigma_n^2}$, the average SNR performance degradation due to channel estimation errors is,

$$\overline{\text{SNR}}_d = 10\log_{10}\left(\frac{\frac{2\sigma^2 P}{\sigma_n^2}(1-\rho_e^2)+1}{\rho_e^2}\right)\text{dB} = 10\log_{10}\left(\frac{\frac{2\sigma^2 P}{\sigma_n^2}(1+\kappa)+1}{\kappa\frac{2\sigma^2 P}{\sigma_n^2}}\right)\text{dB}. \tag{3.80}$$

3.6 Performance Comparisons

In this section, we illustrate our exact analytical results and numerical results to gain insight into the impact of channel estimation errors and feedback delays on performance of the PPC, hybrid and transmit antenna selection methods. We consider a BPSK system with Rayleigh flat-fading channel.

We derived the pdf of the phase difference between the estimated and the predicted

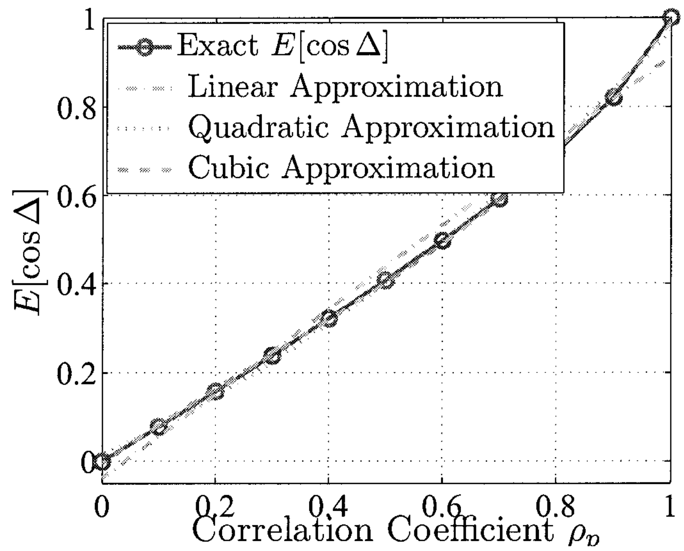
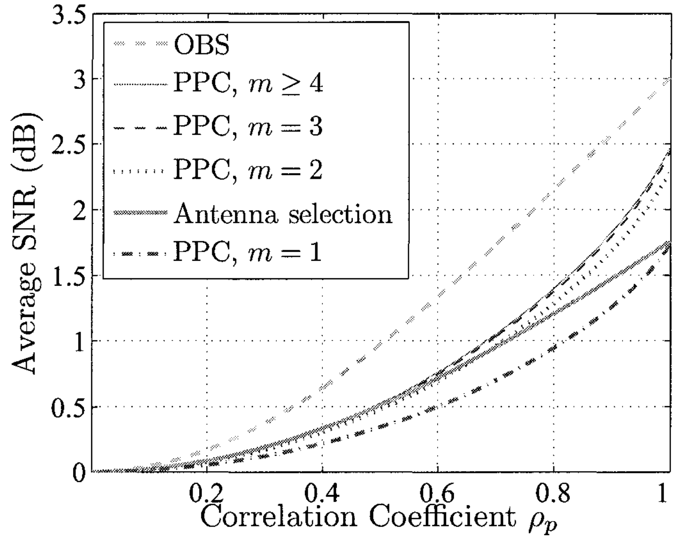


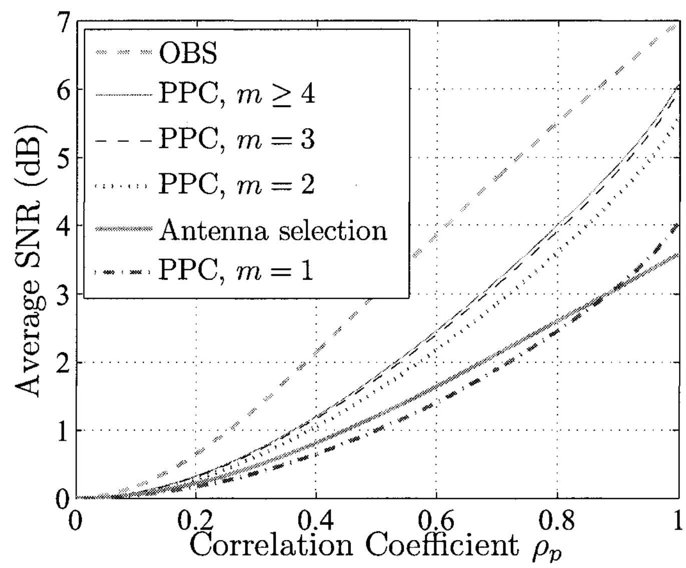
Figure 3.3: The $E[\cos\Delta]$ versus correlation coefficient ρ_p.

channel coefficients in (3.20) and used it to derive the $E[\cos\Delta]$ in (3.28) for analyzing the performance of the PPC and hybrid methods under channel estimation errors and feedback delays. We observe from Figure 3.3 that the $E[\cos\Delta]$ varies almost linearly with ρ_p and has a factor of ρ_p in its expansion (as it is also apparent from its expression in (3.28)).

Figures 3.4 (a) and (b) show the average SNR curves of the transmit antenna selection, PPC and OBS methods in the presence of feedback delay versus correlation coefficient ρ_p for $N = 2$ and $N = 5$ respectively, when the channel estimation is perfect (i.e., $\rho_e = 1$) and $\frac{P}{\sigma_n^2} = 1$. We observe from Figure 3.4 (a) that by using the same number of feedback bits, the antenna selection method performs better than the PPC with $m = 1$ in the presence of feedback delay, while they perform identically when there is no delay ($\rho_p = 1$). For $\rho_p \leq 0.68$, the antenna selection outperforms the 2-bit PPC by using only one feedback bit. This implies that in the presence of feedback



(a) $N = 2$



(b) $N = 5$

Figure 3.4: The average SNR versus correlation coefficient ρ_p, when $\rho_e = 1$ and $\frac{P}{\sigma_n^2} = 1$ for (a) $N = 2$, (b) $N = 5$.

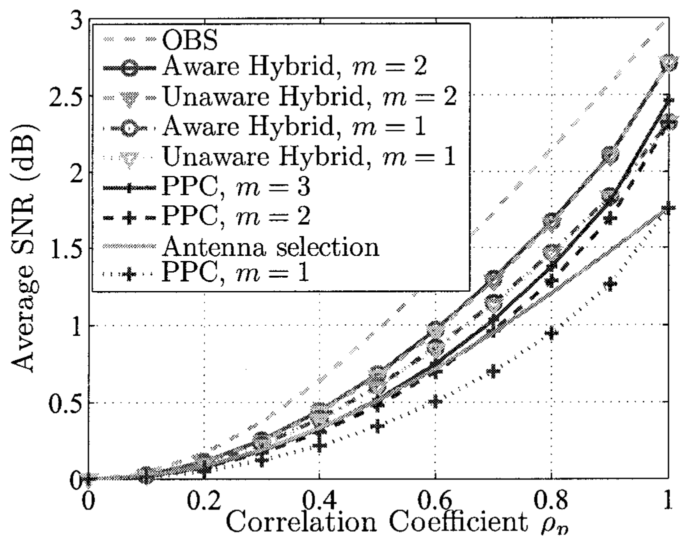


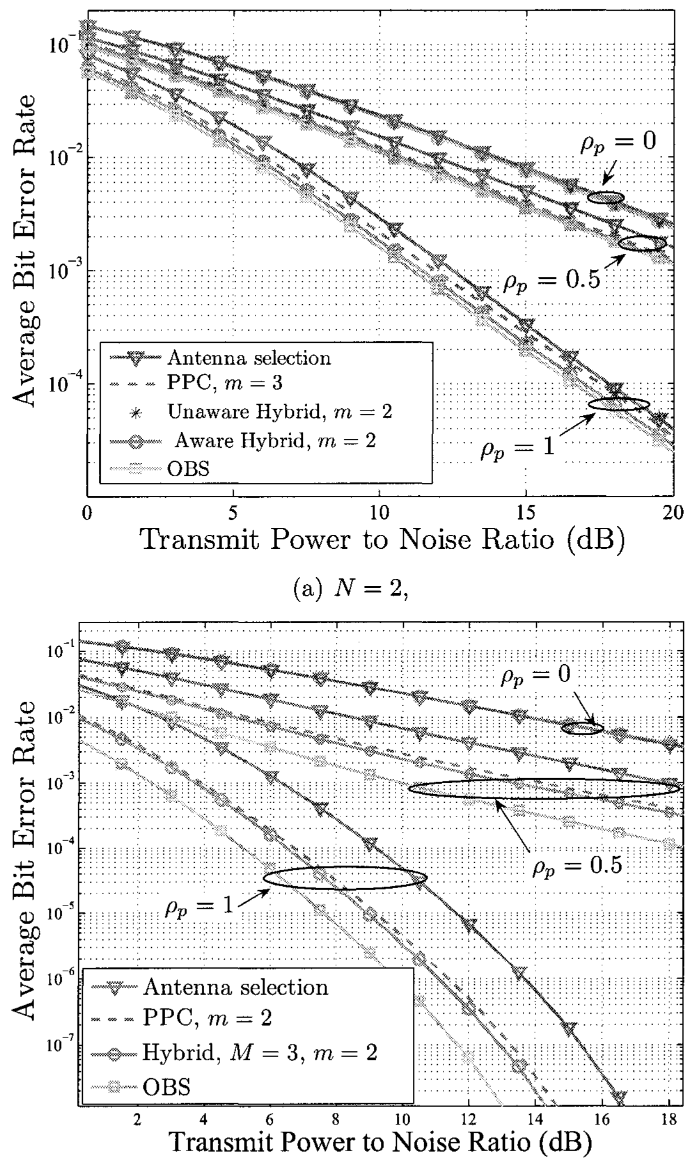
Figure 3.5: The average SNR versus correlation coefficient ρ_p, for $N = 2$, $\rho_e = 1$ and $\frac{P}{\sigma_n^2} = 1$.

delay, using magnitude information is more beneficial than using the same amount of phase information. As m increases, the PPC outperforms the antenna selection and for $m \geq 4$ the performance of PPC remains almost the same. We also observe that the OBS outperforms the antenna selection and PPC method in the presence of feedback delay. For example, if $N = 2$ and $\rho_p = 0.62$, the OBS outperforms the antenna selection with 0.65dB, the 3-bit PPC with 0.61dB and the 1-bit PPC with 0.87dB higher average SNR. As the correlation coefficient ρ_p increases, the performance loss of antenna selection to OBS increases.

We observe in Figure 3.5 that in 2×1 systems with feedback delay, our proposed hybrid method outperforms the PPC method with the same number of feedback bits. We also observe that in the case that two feedback bits are used, the hybrid method outperforms the PPC method in the presence of feedback delay ($\rho_p \neq 1$) while they perform identically when there is no feedback delay ($\rho_p = 1$). For instance, if $\rho_p = 0.7$

the hybrid method achieves 0.18 dB higher average SNR compared with the PPC method using two feedback bits. When three feedback bits are used, the performance gain of hybrid scheme over PPC scheme with the same number of feedback bits increases. For example, if $\rho_p = 0.8$ the hybrid scheme with $m = 2$ attains 0.30 dB gain over the PPC with $m = 3$. For $\rho_p \leq 0.9$, we observe that the hybrid scheme with $m = 1$ outperforms the PPC method with $m = 3$ by using one less feedback bit. These observations again show the superiority of using magnitude information over phase information in the presence of feedback delay. We can also observe that the aware and unaware hybrid methods perform very closely and adapting the hybrid weighting coefficients with feedback delay does not improve the performance.

Figures 3.6 (a) and (b) depict the average BER versus transmit power to noise ratio (TNR) curves for $\rho_p = 0, 0.5, 1$ and perfect channel estimation (i.e., $\rho_e = 1$) when $N = 2$ and $N = 5$ respectively. Figure 3.6 (a) shows that the hybrid method with $m = 2$ outperforms the PPC method with $m = 3$ using the same total number of feedback bits and performs very closely to the OBS in the presence of feedback delay. For example, to achieve an average BER of 10^{-3}, the performance gain of hybrid method over PPC is 0.30dB and its performance loss to OBS is just 0.14 dB if $\rho_p = 0.5$. This implies that when feedback delay occurs, hybrid scheme performs very closely to the OBS by using only three feedback bits and it is more practical to replace the OBS which requires infinite feedback bits with the hybrid scheme. This Figure also shows that the performance of the aware and unaware hybrid methods are almost the same. It is also observed that the feedback delay results in degradation of the diversity gain (the negative slope of the average BER curves) and the coding gain (the gap between the imperfect curves and the perfect curves) of all these schemes.



(a) $N = 2$,

(b) $N = 5$,

Figure 3.6: The average BER versus transmit power to noise ratio (TNR) for different values of ρ_p, when $\rho_e = 1$ and (a) $N = 2$ and (b) $N = 5$.

In the presence of feedback delay ($\rho_p \neq 1$), the diversity gains of all these schemes are almost the same for different values of ρ_p (the diversity gain is around 1) and for ideal feedback case ($\rho_p = 1$) their diversity gain is 2.

Figure 3.6 (b) shows that in the presence of feedback delay ($\rho_p \neq 1$), the diversity gain of the antenna selection, PPC, hybrid and OBS are almost the same (the diversity gain is about 1) and if $\rho_p = 1$, the full diversity gain is achieved (the diversity gain is 5). The feedback delay severely degrades the BER performance of all these schemes by reducing the diversity gain such that for $\rho_p = 0$ all these methods have the same BER curves as SISO case. For example, if $\rho_p = 0.5$ the hybrid scheme experiences performance loss of 11.6 dB at the average BER of 10^{-3} compared to $\rho_p = 1$. We also observe that the proposed hybrid scheme with $M = 3$ and $m = 2$ outperforms the PPC method with $m = 2$ using the same number of feedback bits with feedback delay. For instance, if $\rho_p = 0.5$ the hybrid scheme achieves 0.28dB performance gain over the PPC method at the average BER of 10^{-3}. By comparing Figures 3.6 (a) and (b) we observe that the performance loss due to feedback delay increases as the number of transmit antennas becomes larger. For example, if $\rho_p = 0.5$ the performance loss of antenna selection method at the average BER of 10^{-3} increases from 9dB to 12dB as the number of transmit antennas increases from 2 to 5.

Figures 3.7 (a) and (b) show the average BER curves versus TNR (i.e., $\frac{P}{\sigma_n^2}$) in the presence of channel estimation errors and no feedback delay (i.e., $\rho_p = 1$) for $\rho_e = 0.9$, $\sqrt{\frac{0.5P}{0.5P+\sigma_n^2}}$ and 1, in respectively 2 and 5 transmit antenna systems.

We observe that if ρ_e is fixed (e.g. $\rho_e = 0.9$), error floor happens and the performance is severely degraded. If ρ_e increases with TNR (e.g., $\rho_e = \sqrt{\frac{0.5P}{0.5P+\sigma_n^2}}$), error floor does not happen and the performance degradation is less severe. We observe

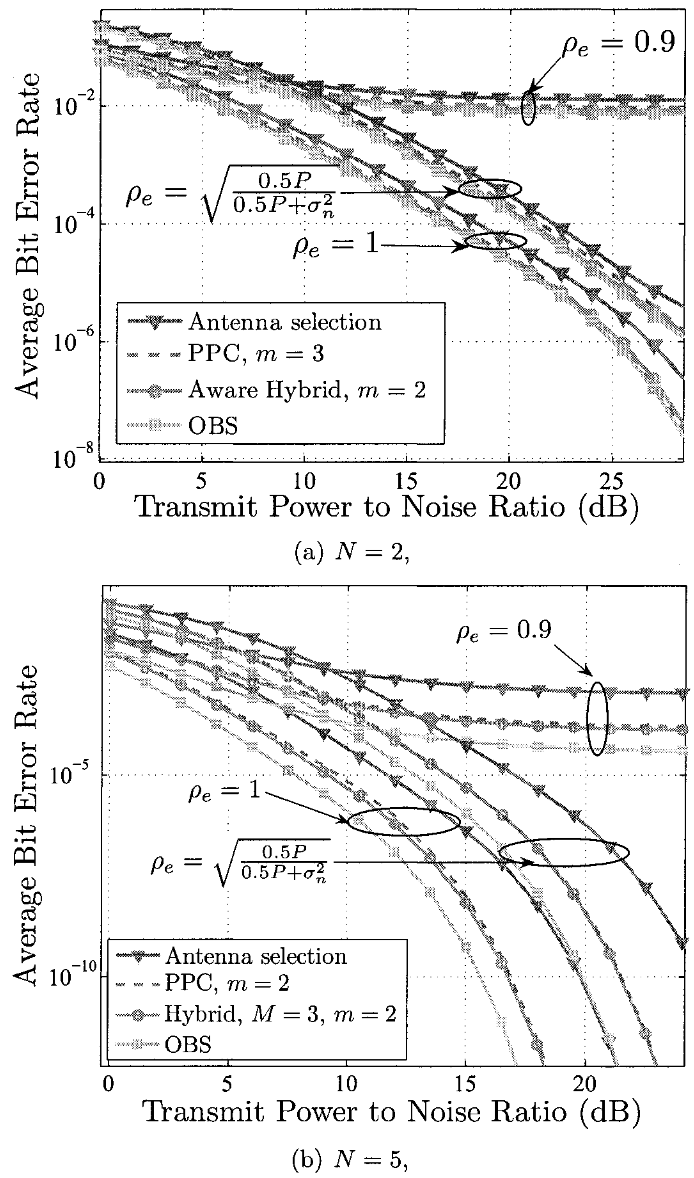


Figure 3.7: The average BER versus TNR for different values of ρ_e, when $\rho_p = 1$, $2\sigma^2 = 1$ and (a) $N = 2$ and (b) $N = 5$.

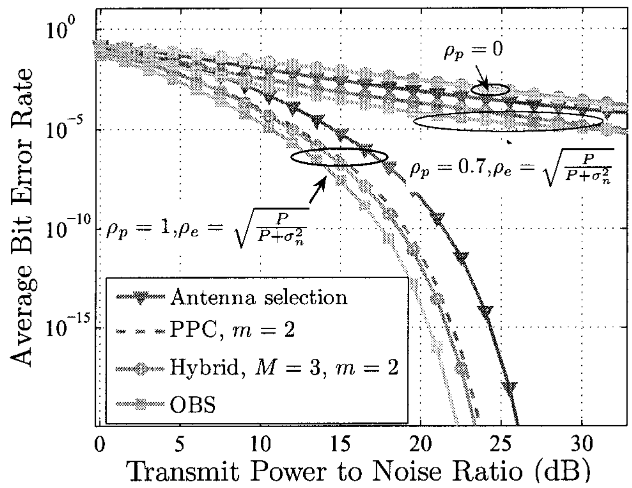


Figure 3.8: The average BER versus TNR for different values of ρ_e and ρ_p, for $N = 5$ and $2\sigma^2 = 1$.

that if $\rho_e = \sqrt{\frac{0.5P}{0.5P+\sigma_n^2}}$ (i.e., $\kappa = 0.5$), the diversity gain (the negative slope of the average BER curves) is the same as the perfect estimation case (i.e., $\rho_e = 1$) and the performance degradation is due to reduction in the coding gain (the horizontal shift of the average BER curves). The performance comparisons of different schemes in the presence of channel estimation errors are similar to performance comparisons in the presence of feedback delay (see observations for Figure 3.6).

Figure 3.8 depicts the combined impact of channel estimation errors and feedback delay on the average BER performance of several limited-feedback transmit diversity schemes in 5×1 systems with $2\sigma^2 = 1$. If ρ_e is fixed (e.g. $\rho_e = 0.7$), error floors are observed and the impact of feedback delay (i.e., ρ_p) becomes less significant. If ρ_e increases with TNR (e.g. $\rho_e = \sqrt{\frac{\kappa P}{\kappa P+\sigma_n^2}}$), we observe that the impact of feedback delay (ρ_p) becomes more significant. For example, if $\rho_e = \sqrt{\frac{\kappa P}{\kappa P+\sigma_n^2}}$ with $\kappa = 1$ we observe that by reducing ρ_p from 1 to 0.7 the diversity gain becomes less resulting in

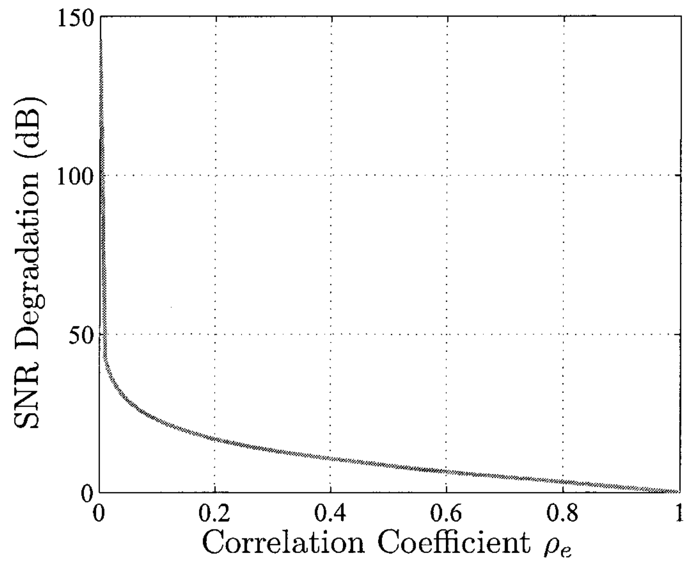


Figure 3.9: The average SNR degradation due to channel estimation errors versus fixed correlation coefficient ρ_e, for $\frac{2\sigma^2 P}{\sigma_n^2} = 1$.

severe BER performance degradations. We observe that if $\rho_p = 0.7$ the diversity gain becomes the same as the case that $\rho_p = 0$ which is equal to one (the same as SISO case).

The average SNR degradation due to channel estimation errors (compared to the perfect channel estimation case), $\overline{\text{SNR}}_d$, is plotted versus fixed ρ_e in Figure 3.9 for $\frac{2\sigma^2 P}{\sigma_n^2} = 1$. We observe that the average SNR degradation increases severely as $\rho_e \to 0$. As the channel estimation errors and feedback delay have separable impacts on the SNR performance, the average SNR curves for different values of ρ_e and ρ_p can be easily obtained by subtracting the average SNR degradation in Figure 3.9 from the average SNRs in Figures 3.4 and 3.5 for different values of ρ_e. The average SNR degradation due to channel estimation errors with $\rho_e = \sqrt{\frac{2\sigma^2 \kappa P}{2\sigma^2 \kappa P + \sigma_n^2}}$ is depicted as a function of TNR (i.e., $\frac{P}{\sigma_n^2}$) in Figure 3.10 for different values of κ and $2\sigma^2 = 1$. We

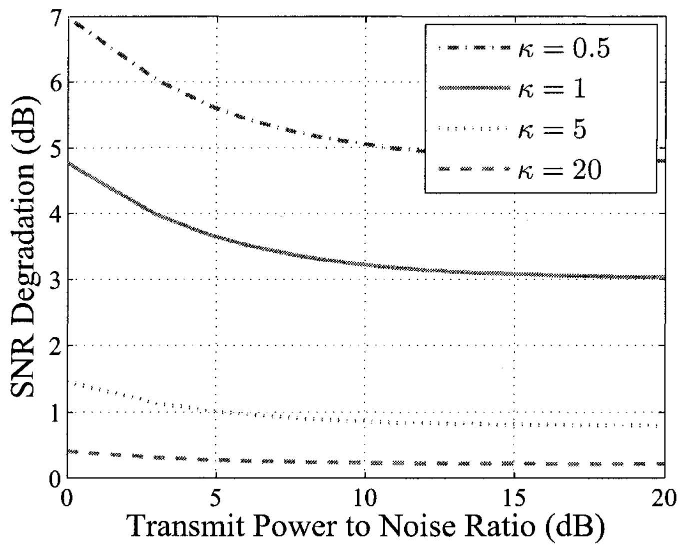


Figure 3.10: The average SNR degradation due to channel estimation errors, $\overline{\mathrm{SNR}}_d$, versus transmit power to noise ratio , $\frac{P}{\sigma_n^2}$, for different values of κ when $2\sigma^2 = 1$.

observe that as the transmit power to noise ratio increases, the performance loss due to channel estimation errors decreases. We also observe that as κ increases, the average SNR degradation due to channel estimation errors decreases and the dependability of the SNR degradation to the TNR becomes less.

3.7 Summary and Conclusions

In this paper we analyzed the performance of the PPC, the hybrid and the antenna selection methods in the presence of channel estimation errors and feedback delay. We derived the pdf of phase difference between the estimated and predicted channel coefficients. Using this pdf, we derived the exact average SNR for PPC in the presence of channel estimation errors and feedback delay. We analyzed the performance of hybrid method for 2×1 in two cases where the correlation coefficient between the estimated and predicted subchannels is known or unknown to the transmitter. We derived the optimal hybrid weighting coefficients when the correlation coefficient is known and derived the average SNR expression in this case. Deriving the average SNR when the correlation coefficient is unknown, we observed that adapting the hybrid weighting coefficients to feedback delay does not improve the performance noticeably. We derived the pdf of received SNR and the exact closed-form expression for average SNR and BER for the antenna selection method in the presence of channel estimation errors and feedback delay in 2×1 systems.

We showed that channel estimation errors and feedback delays have separable impacts on the average SNR performance. We also showed that the average SNR degradation due to channel estimation errors compared to the perfect estimation case is identical in these methods. We observed that fixed estimation errors result in error floors and feedback delay does not effect the performance significantly. However in practice, where estimation error decreases with SNR, we showed that the diversity gain does not change under estimation errors and remains the same as the perfect estimation case. In this case we showed that the performance degradation under channel estimation errors is due to the reduction of coding gain. We also showed that if the estimation error depends on the SNR, the feedback delay results in significant performance degradations by reducing the diversity gain. If the channel estimation is perfect, we showed that the feedback delay results in reduction of diversity gain to one (the same as SISO) and as a result, severe performance degradations. As the number of transmit antennas increases, this degradations become more severe. We showed that our proposed hybrid method outperforms the PPC and antenna selection methods in the presence of feedback delay and channel estimation errors. We observed that in 2×1 systems with feedback delay the performance of antenna selection is better than the 1-bit PPC, while they perform the same when there is no channel error. We also observe that the 2-bit feedback hybrid scheme outperforms the 2-bit PPC under channel errors, while they perform the same when there is no channel errors. This implies that employing only one bit magnitude information is more advantageous than one bit phase information under channel estimation errors and feedback delay. We showed that in the presence of channel errors, the proposed hybrid method outperforms the PPC using the same number of bits and performs very closely to the OBS by using only three feedback bits (e.g., as close as 0.14dB at the average BER of 10^{-3} when $\rho_p = 0.5$ and $\rho_e = 1$).

Chapter 4

Conclusions and Future Work

4.1 Conclusion

Simple practical transmit diversity schemes with very low feedback rates are of great importance to the downlink of wireless communication systems. In other hands due to the limited bandwidth of the feedback link it is practically important to evaluate the impact and possible forms of channel information. In order to gain insight to the value and efficiency of different kinds of CSI we consider the antenna selection and the PPC methods where the latter only applies the phase information and the former applies only the magnitude information of the channel. We also propose a simple practical hybrid method which applies both the phase and magnitude information of the channel and outperforms the practical PPC and antenna selection methods. For 2-transmit antenna systems, this scheme applies one-bit magnitude information and outperforms the PPC method with the same number of feedback bits and performs very close to the OBS using very few bits of feedback. This implies that only one bit magnitude information is enough to elevate the performance of PPC method very close to the optimal beamforming. For N-transmit antennas systems, the proposed hybrid method steers the phase of the stronger subchannels which results in potential better performance compared to the PPC and the EGT by avoiding power dissipation over insignificant subchannels. The hybrid method requires considerably reduced electronic circuitry and less computational complexity compared with the PPC and EGT methods as it performs PPC only on selected subchannels. Our results for Rayleigh channels reveal that selection of 75% of antenna elements results in best performance when beamformer only adjusts the phase of transmit signals. In order to achieve a close to optimal performance and more reduced transmitter complexity, we suggest to use only about half of transmit antennas.

The antenna selection scheme as a simple solution to the high implementation costs of MIMO systems, suffers from a degraded performance in correlated Rayleigh and Rician fading channels. As a practical solution to this, we present the transform domain selection and specifically the FFT-based selection method which substantially outperforms the antenna selection in correlated Rayleigh channels and systems applying linear arrays of omnidirectional antennas. In order to reduce the computational complexity of the PPC method, we propose an almost-optimal fast procedure for finding the optimal PPC coefficients. This procedure reduces the computational complexity order of the finding optimal PPC codewords from $O\left(2^{m(N-1)}\right)$ to $O\left(N-1\right)$.

To incorporate more practical issues, we analyze the impact of channel estimation errors and feedback delays on the performance of the PPC, hybrid and transmit antenna schemes. We derive the average SNR expression for the PPC method in the presence of channel estimation errors and feedback delay and derive the probability distribution of the phase difference between the estimated and predicted subchannels.

We analyze the SNR performance of the hybrid method under imperfect channel estimations and feedback delays. We derive the average SNR of hybrid method in two cases that the correlation coefficient between the estimated and predicted subchannels is known or unknown to the transmitter and optimize the hybrid coefficients if this correlation coefficient is available. We also analyze the performance of transmit antenna selection method in the presence of feedback delay and derive the exact closed-form expression for the average SNR and average BER of this method in 2×1 systems. For all these methods we show that the channel estimation errors and feedback delay have separable impacts on the average SNR performance, and the average SNR degradation due to the channel estimation errors is identical in these methods. Using our analytical and numerical results, we present a useful performance evaluation and comparison of these important practical methods in the presence of channel estimation errors and feedback delay. We show that our proposed hybrid scheme outperforms the PPC and antenna selection methods in the presence of channel errors and performs very closely to the OBS by using very few bits of feedback. We also show that methods using magnitude information (such as antenna selection and hybrid methods) perform better than the methods using only phase information (such as PPC method) in the presence of feedback delay. This implies superiority of employing magnitude information over employing phase information in case of experiencing feedback delay. We show that if the estimation error is invariant to SNR error floors happen. For the realistic assumption that the channel estimation error decreases with SNR, we show that the channel estimation errors does not change the diversity gain compared to the perfect channel estimation case and the performance degradation is due to reduction of coding gain by an identical factor for all methods. In this case we also show that feedback delay results in severe performance degradation by reducing the diversity gain as low as one (the same as SISO case).

4.2 Future Work

Simple limited feedback schemes with small number of antennas at the mobile set and low receiver complexity as well as a reduced number of RF chains are desirable for downlink transmission due to the size and power limitations of most of the current handheld devices. Some current, and most future, base stations are expected to be equipped with at least two transmit antennas in each sector that can be used for downlink transmit diversity. In this thesis we proposed a simple yet efficient hybrid scheme which can achieve near optimal performance using few feedback bits. A possible expansion of our work could be to study the impact of hybrid method on the multiuser diversity gain and throughput of multiuser wireless systems. Furthermore, the study of interaction between transmit diversity and the multiuser diversity that already exists in scheduled systems is suggested as a possible future work.

We studied the impact of channel estimation errors and feedback delay on several practical transmit diversity schemes. Analyzing the joint and separate impact of channel estimation errors and feedback delays on the performance of more complicated beamforming schemes employing vector quantization techniques such as the ones in [9, 10] are also considered as possible future works. Another possible extension of the work in this thesis is to analyze different limited-feedback transmit diversity schemes (such as the PPC, hybrid and transmit antenna selection schemes) in the presence of other channel errors and feedback impairments, such as feedback link error.

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Appendix A

Derivation of Average SNR in Antenna Selection Scheme

In this Appendix we derive (2.15). The average SNR in the antenna selection method is given by,

$$\overline{\text{SNR}} = \frac{P}{\sigma^2} E\left[\left(\max_{i=1,\ldots,N} |h_i|\right)^2\right]. \tag{A.1}$$

The cumulative distribution function (cdf) of the random variable $Z \triangleq \max_{i=1,\ldots,N} |h_i|$, where $|h_1|, \ldots, |h_N|$ are i.i.d Rayleigh random variables, is as follows

$$\begin{aligned} F_Z(z) &= Pr\left(|h_1|, |h_2|, \ldots, |h_N| \leq z\right) \\ &= F_{|h_1|}(z) \ldots F_{|h_N|}(z) = F_{|h|}^N(z). \end{aligned} \tag{A.2}$$

The last two equations in (A.2) are derived using the fact that $|h_1|, \ldots, |h_N|$ are i.i.d Rayleigh random variables. By taking derivative from (A.2) we have pdf of Z as

$$f_Z(z) = N F_{|h|}^{N-1}(z) f_{|h|}(z) = 2N(1 - e^{-z^2})^{N-1} z e^{-z^2} u(z), \tag{A.3}$$

where $u(z)$ is the unit step function. Using (A.3), the average SNR is derived as,

$$\begin{aligned}
\overline{\mathrm{SNR}} &= \frac{P}{\sigma^2}\int_0^\infty 2Nz^3(1-e^{-z^2})^{N-1}e^{-z^2}dz \\
&= \frac{P}{\sigma^2}\int_0^\infty Nx\sum_{k=0}^{N-1}\binom{N-1}{k}(-e^{-x})^k e^{-x}dx \\
&= \frac{NP}{\sigma^2}\sum_{k=0}^{N-1}(-1)^k\binom{N-1}{k}\int_0^\infty xe^{-(k+1)x}dx \\
&= \frac{P}{\sigma^2}\sum_{k=0}^{N-1}\binom{N}{k+1}\frac{(-1)^k}{(k+1)}. \qquad (A.4)
\end{aligned}$$

Appendix B

Derivation of Average SNR in PPC method

In this Appendix we derive (2.20). For the PPC with the first coefficient method the average SNR is given by

$$\begin{aligned}\overline{\mathrm{SNR}} &= \frac{P}{N\sigma^2}E\left[\left|\sum_{i=1}^{N} h_i e^{jQ(\angle h_1 - \angle h_i)}\right|^2\right] \\ &= \frac{P}{N\sigma^2}E\left[\sum_{p=1}^{N}\sum_{q=1}^{N} h_p h_q^* e^{jQ(\angle h_1 - \angle h_p)} e^{-jQ(\angle h_1 - \angle h_q)}\right] \\ &= \frac{P}{N\sigma^2}E\left[\sum_{p=1}^{N}\sum_{q=1}^{N} |h_p||h_q| e^{j\theta_p} e^{-j\theta_q}\right]. \end{aligned} \tag{B.1}$$

Where θ_p and θ_q are respectively quantization error of $\angle h_p - \angle h_1$ and $\angle h_q - \angle h_1$. Using this, average SNR is given by

$$\begin{aligned}\overline{\mathrm{SNR}} &= \frac{P}{N\sigma^2}E\Big[\sum_{p=q=1}^{N} |h_p|^2 + \sum_{p=2}^{N} |h_p||h_1| e^{j\theta_p} \\ &\quad + \sum_{q=2}^{N} |h_1||h_q| e^{-j\theta_q} + \sum_{p\neq q=2}^{N}\sum_{q=2}^{N} |h_p||h_q| e^{j\theta_p} e^{-j\theta_q}\Big]. \end{aligned} \tag{B.2}$$

Using the fact that θ_p and θ_q are uniformly distributed over $\left(-\frac{\pi}{2^m}, \frac{\pi}{2^m}\right)$ and are uncorrelated with $|h_i|$'s, we can simply get (2.20) from (B.2) as follows,

$$\begin{aligned}\overline{\mathrm{SNR}} &= \frac{P}{N\sigma^2}\Big[NE\left[|h_1|^2\right] + 2(N-1)\left(E|h_1|\right)^2 E[\cos\theta] \\ &\quad +(N-1)(N-2)\left(E|h_1|\right)^2\left(E[\cos\theta]\right)^2\Big], \qquad \text{(B.3)} \\ &= \frac{P}{N\sigma_n^2}\left[N + (N-1)2^{m-1}\sin\frac{\pi}{2^m} + (N-1)(N-2)\frac{2^{2m-2}}{\pi}\sin^2\frac{\pi}{2^m}\right]. \qquad \text{(B.4)}\end{aligned}$$

Appendix C

Derivation of g_1 and g_2 in Hybrid Method

In hybrid method average SNR is given by:

$$\begin{aligned}\overline{\text{SNR}} &= \max_{g_1,\ g_2} E\left[\left|g_1 h_1 + g_2 h_2 e^{jQ(\angle h_1 - \angle h_2)}\right|^2 \Big| |h_1| > |h_2|\right] \frac{P}{\sigma_n^2} \\ &= \max_{g_1,\ g_2} E\left[{g_1}^2 |h_1|^2 + {g_2}^2 |h_2|^2 + 2 g_1 g_2 |h_1||h_2| \cos\theta \Big| |h_1| > |h_2|\right] \frac{P}{\sigma_n^2} \qquad \text{(C.1)}\end{aligned}$$

where θ is the quantization error of $\angle h_1 - \angle h_2$ and is uniformly distributed over $(-\frac{\pi}{2^m}, \frac{\pi}{2^m})$ and is uncorrelated from $|h_1||h_2|$ given $|h_1| > |h_2|$. Using the joint pdf of $|h_1|$ and $|h_2|$ given $|h_1| > |h_2|$ as,

$$f_{|h_1|,|h_2| \big| |h_1|>|h_2|}(r_1, r_2) = \begin{cases} 8 r_1 r_2 e^{-(r_1^2 + r_2^2)} & \text{if } r_1 > r_2 \geq 0 \\ 0 & \text{Otherwise} \end{cases} \qquad \text{(C.2)}$$

we calculate the conditional expectations in (C.1)as, $E[|h_1|^2 \big| |h_1| > |h_2|] = \frac{3}{2}$, $E[|h_2|^2 \big| |h_1| > |h_2|] = \frac{1}{2}$ and $E[|h_1||h_2| \big| |h_1| > |h_2|] = \frac{\pi}{4}$ and we derive average SNR as follows

$$\overline{\text{SNR}} = \max_{g_1,\ g_2; g_1^2 + g_2^2 = 1, g_1, g_2 \in \mathcal{R}^+} \frac{P}{\sigma_n^2} \left(\frac{3}{2} g_1^2 + \frac{1}{2} g_2^2 + g_1 g_2 2^{m-1} \sin\frac{\pi}{2^m}\right). \qquad \text{(C.3)}$$

By applying Lagrange multipliers subject to the constraint of $g_1^2 + g_2^2 = 1$, g_1, g_2 are found as (2.28) and (2.29). Consequently, the maximized average SNR is derived as,

$$\overline{\text{SNR}} = \frac{P}{\sigma_n^2}\left[1 + \frac{1}{2}\sqrt{1 + 2^{2m-2}\sin^2\frac{\pi}{2^m}}\right]. \tag{C.4}$$

Appendix D

Derivation of Average BER in General

In systems using BPSK or QPSK with Gray coding the BER can be exactly expressed as,

$$P_e = \frac{1}{2}E[\text{erfc}(\sqrt{\gamma})] = \int_0^\infty \frac{1}{2}\text{erfc}(\sqrt{\gamma})f_\gamma(\gamma)d\gamma, \quad \text{(D.1)}$$

where $\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,dt$ is the complementary error function and γ is the SNR per bit.

Lemma 1. *Consider systems using BPSK or QPSK with Gray coding and with SNR per bit of* $\gamma = \frac{ZP}{\sigma_n^2 \log_2 D}$, *where* Z *is a function of channel gain and* D *is the number of constellation points. If* $F_Z(0) = 0$, *where* $F_Z(z)$ *is the cumulative distribution function (cdf) of random variable* Z, *average bit error rate is derived as,*

$$P_e = \frac{\sqrt{\beta}}{2\sqrt{2\pi}}\int_0^\infty z^{-\frac{1}{2}}e^{-\frac{\beta z}{2}}F_Z(z)dz, \quad \text{(D.2)}$$

where $\beta = \frac{2P}{\sigma_n^2 \log_2 D}$.

Proof. Using (D.1), we have

$$\begin{aligned}
P_e &= \frac{1}{2}E\left[\operatorname{erfc}\left(\sqrt{\frac{P}{\sigma^2\log_2 D}Z}\right)\right] = \frac{1}{2}\int_0^\infty \operatorname{erfc}\sqrt{\frac{\beta z}{2}} f_Z(z)dz \\
&= \frac{1}{2}\left[\operatorname{erfc}\left(\sqrt{\frac{\beta z}{2}}\right)F_Z(z)\right]\Big|_0^\infty - \frac{1}{2}\int_0^\infty F_Z(z)d\left(\operatorname{erfc}(\sqrt{\frac{\beta z}{2}})\right) \\
&= -\frac{1}{2}\int_0^\infty F_Z(z)d\left(\operatorname{erfc}(\sqrt{\frac{\beta z}{2}})\right) \qquad \text{(D.3)} \\
&= \frac{\sqrt{\beta}}{2\sqrt{2\pi}}\int_0^\infty z^{-\frac{1}{2}}e^{-\frac{\beta z}{2}}F_Z(z)dz, \qquad \text{(D.4)}
\end{aligned}$$

where (D.3) holds because in these systems $F_Z(0) = 0$, thus $\operatorname{erfc}(\sqrt{\frac{\beta z}{2}})F_Z(z)\big|_0^\infty = 0$. □

Appendix E

Average BER in Antenna Selection

The SNR in antenna selection method is given by,

$$\text{SNR} \;=\; \frac{P}{\sigma^2} \max_{i=1,\ldots,N} |h_i|^2. \tag{E.1}$$

The cdf of random variable $Z \triangleq \max_{i=1,\ldots,N} |h_i|^2$ is

$$F_Z(z) \;=\; F^N_{|h|^2}(z) = (1 - e^{-z})^N u(z). \tag{E.2}$$

This cdf satisfies the condition of Lemma 1 in Appendix D (i.e. $F_Z(0) = 0$). By using Lemma 1 and the definition of Gamma function, $\Gamma(n+1) = b^{n+1} \int_0^\infty x^n e^{-bx} dx$, and also doing some manipulations the exact average BER is derived as,

$$P_e = \frac{1}{2} \sum_{i=0}^{N} (-1)^i \binom{N}{i} \sqrt{\frac{\beta}{\beta + 2i}}. \tag{E.3}$$

By using the above result, when $N = 1$ the average BER is given by,

$$P_e = \frac{1}{2} \left(1 - 2\sqrt{\frac{\beta}{\beta+2}} + \sqrt{\frac{\beta}{\beta+4}} \right). \tag{E.4}$$

Average BER expression of the PPC method using one bit feedback is exactly the same as (E.4).

Appendix F

Average BER in Optimal Beamforming Scheme

In the optimal beamforming method for $N \times 1$ wireless systems, the SNR is $\frac{P(\|\mathbf{h}\|^2)}{\sigma^2}$ where $Z \triangleq \|\mathbf{h}\|^2$ has a central chi-squared distribution with $2N$ degrees of freedom. The cdf of Z is given by,

$$F_Z(z) = \left(1 - \sum_{i=0}^{N-1} \frac{z^i e^{-z}}{i!}\right) u(z). \tag{F.1}$$

As $F_Z(0) = 0$, Lemma 1 in Appendix D can be applied here. By doing some manipulations and using the definition of Gamma function, the exact closed-form average BER expression is reduced to

$$P_e = \frac{1}{2}\left(1 - \sum_{i=0}^{N-1} \frac{(2i)!}{2^i (i!)^2} \frac{\sqrt{\beta}}{(\beta+2)^{i+\frac{1}{2}}}\right). \tag{F.2}$$

Appendix G

Lemma 2

Lemma 2. *If X is a zero-mean circularly symmetric complex Gaussian (CSCG) random variable with variance of σ_x^2, and S is a complex random variable independent of X, with deterministic magnitude (i.e., $|S|$ =constant), then SX is a zero-mean CSCG random variable with variance of $|S|^2\sigma_x^2$.*

Appendix H

Derivation of $f_{X_1}(x_1)$

In order to derive the pdf of X_1 from

$$f_{X_1}(x_1) = \int_{x_2=0}^{\infty}\int_{y_1=0}^{\infty}\int_{y_2=0}^{y_1} f_{X_1,X_2,Y_1,Y_2}(x_1,x_2,y_1,y_2)\,dy_2\,dy_1\,dx_2 \tag{H.1}$$

we use the Taylor expansion of the Bessel function, $I_0(x)=\sum_{k=0}^{\infty}\frac{(\frac{x}{2})^{2k}}{(k!)^2}$ in the expression of $f_{X_1,X_2,Y_1,Y_2}(x_1,x_2,y_1,y_2)$ in (3.69) and take the integrations as follows,

$$\begin{aligned} f_{X_1}(x_1) &= \int_{x_2=0}^{\infty}\int_{y_1=0}^{\infty}\int_{y_2=0}^{y_1} \frac{2x_1x_2y_1y_2\exp\left(-\frac{x_1^2+x_2^2+y_1^2+y_2^2}{2\sigma^2(1-\rho_p^2)}\right)}{\sigma^8(1-\rho_p^2)^2} \\ &\quad\times\sum_{k_1=0}^{\infty}\frac{\left(\frac{\rho_p x_1 y_1}{2\sigma^2(1-\rho_p^2)}\right)^{2k_1}}{(k_1!)^2}\sum_{k_2=0}^{\infty}\frac{\left(\frac{\rho_p x_2 y_2}{2\sigma^2(1-\rho_p^2)}\right)^{2k_2}}{(k_2!)^2}dy_2\,dy_1\,dx_2 \qquad \text{(H.2)} \\ &\qquad\qquad \text{(H.3)} \\ &= \int_{x_2=0}^{\infty}\int_{y_1=0}^{\infty}\frac{16\sqrt{2}(1-\rho_p^2)^{\frac{3}{2}}}{\sigma}e^{-\left({x'_1}^2+{x'_2}^2+{y'_1}^2\right)}\sum_{k_1=0}^{\infty}\frac{\rho_p^{2k_1}(x'_1y'_1)^{2k_1+1}}{(k_1!)^2} \\ &\quad\times\sum_{k_2=0}^{\infty}\frac{{\rho_p}^{2k_2}\,(x'_2)^{2k_2+1}}{k_2!}\left(1-\sum_{q_2=0}^{k_2}\frac{\exp\left(-{y'_1}^2\right){y'_1}^{2q_2}}{q_2!}\right)dy'_1\,dx'_2 \qquad \text{(H.4)} \end{aligned}$$

for $x_1 \geq 0$ and zero elsewhere, where $x'_i \triangleq \frac{x_i}{\sigma\sqrt{2(1-\rho_p^2)}}$ and $y'_i \triangleq \frac{y_i}{\sigma\sqrt{2(1-\rho_p^2)}}$ for $i=1,2$ and equation (H.4) is derived from the integral $\int x^{2n+1}e^{-x^2}dx = -\frac{n!}{2}e^{-x^2}\sum_{k=0}^{n}\frac{x^{2k}}{k!}$.

By using the integral $\int_0^\infty x^{2n+1}\exp(-\frac{x^2}{2\sigma^2})dx = 2^n n!\sigma^{2n+2}$, $f_{X_1}(x_1)$ is derived as,

$$\begin{aligned} f_{X_1}(x_1) &= \sum_{k_2=0}^{\infty} \frac{8\sqrt{2(1-\rho_p^2)}\rho_p^{2k_2}}{\sigma^3 k_2!} \int_{x_2'=0}^{\infty} \exp\left(-{x_2'}^2\right) {x_2'}^{2k_2+1} dx_2' \\ &\times \int_{y_1=0}^{\infty} \sum_{k_1=0}^{\infty} \frac{(x_1' y_1')^{2k_1+1} e^{-({x_1'}^2+{y_1'}^2)}}{(k_1!)^2} \left(1 - \sum_{q_2=0}^{k_2} \frac{e^{-{y_1'}^2} {y_1'}^{2q_2}}{q_2!}\right) dy_1' \quad \text{(H.5)} \\ &= \sum_{k_2=0}^{\infty}\sum_{k_1=0}^{\infty} \frac{4\sqrt{2}(1-\rho_p^2)^{\frac{3}{2}}\rho_p^{2k_1+2k_2}}{\sigma k_1!} \left(\frac{x_1}{\sigma\sqrt{2(1-\rho_p^2)}}\right)^{2k_1+1} \\ &\times \exp\left(-\frac{x_1^2}{2\sigma^2(1-\rho_p^2)}\right) \left[1 - \sum_{q_2=0}^{k_2} \binom{k_1+q_2}{k_1} \frac{1}{2^{k_1+q_2+1}}\right], \quad \text{(H.6)} \end{aligned}$$

for $x_1 \geq 0$ and zero elsewhere.

We simplify the pdf expression in H.6 using the geometric series property that $\sum_{k=0}^{\infty} R^k = \frac{1}{1-R}$ for $|R| < 1$ as follows,

$$\begin{aligned} f_{X_1}(x_1) &= \sum_{k_2=0}^{\infty}\sum_{k_1=0}^{\infty} \frac{4\sqrt{2}(1-\rho_p^2)^{\frac{3}{2}}\rho_p^{2k_1+2k_2}}{\sigma k_1!} {x_1'}^{2k_1+1} \\ &\times \exp\left(-{x_1'}^2\right) \left[1 - \sum_{q_2=0}^{k_2} \binom{k_1+q_2}{k_1} \frac{1}{2^{k_1+q_2+1}}\right] u(x_1) \quad \text{(H.7)} \\ &= \frac{4x_1(1-\rho_p^2)}{\sigma^2} \exp\left(-{x_1'}^2\right) \left[\sum_{k_2=0}^{\infty} \rho_p^{2k_2} \sum_{k_1=0}^{\infty} \frac{1}{k_1!} (\rho_p x_1')^{2k_1} \right. \\ &\left. - \sum_{k_1=0}^{\infty} \frac{(\rho_p x_1')^{2k_1}}{2k_1!(1-\rho_p^2)} \sum_{q_2=0}^{\infty} \binom{k_1+q_2}{k_1} \sum_{k_2=q_2}^{\infty} \frac{\rho_p^{2k_2}}{2^{q_2}} \right] u(x_1) \quad \text{(H.8)} \\ &= \frac{4x_1 e^{-{x_1'}^2}}{\sigma^2} \left[e^{\rho_p^2 {x_1'}^2} - \sum_{k_1=0}^{\infty} \frac{\left(\frac{\rho_p^2 {x_1'}^2}{(1-\frac{\rho_p^2}{2})}\right)^{k_1}}{2(1-\frac{\rho_p^2}{2})k_1!} \right. \\ &\left. \times \sum_{q_2=0}^{\infty} \binom{k_1+q_2}{k_1} \left(\frac{\rho_p^2}{2}\right)^{q_2} \left(1-\frac{\rho_p^2}{2}\right)^{k_1+1} \right] u(x_1), \quad \text{(H.9)} \end{aligned}$$

where $u(.)$ denotes the unit step function. By applying the fact that $\sum_{q_2=0}^{\infty} \binom{k_1+q_2}{k_1} \left(\frac{\rho_p^2}{2}\right)^{q_2} (1-$

$\frac{\rho_p^2}{2})^{k_1+1} = 1$, where $f(k_1+1; q_2, \frac{\rho_p^2}{2}) = \binom{k_1+q_2}{k_1}\left(\frac{\rho_p^2}{2}\right)^{q_2}(1-\frac{\rho_p^2}{2})^{k_1+1}$ is a negative binomial distribution, we derive the exact closed-form expression for $f_{X_1}(x_1)$ as follows,

$$f_{X_1}(x_1) = \frac{4x_1 e^{-{x'_1}^2}}{\sigma^2}\left[\exp\left(\frac{\rho_p^2 {x'_1}^2}{2}\right) - \sum_{k_1=0}^{\infty}\frac{\left(\frac{\rho_p^2 {x'_1}^2}{2(1-\frac{\rho_p^2}{2})}\right)^{k_1}}{2k_1!(1-\frac{\rho_p^2}{2})}\right]u(x_1) \quad \text{(H.10)}$$

$$= \frac{2x_1}{\sigma^2}\left[\exp\left(-\frac{x_1^2}{2\sigma^2}\right) - \frac{\exp\left(-\frac{x_1^2}{2\sigma^2(1-\frac{\rho_p^2}{2})}\right)}{2(1-\frac{\rho_p^2}{2})}\right]u(x_1). \quad \text{(H.11)}$$

Appendix I

Derivation of Diversity Gain and Coding Gain in General

The coding gain G_c determines the horizontal shift of the BER curve and the diversity gain G_d determines the slope of the BER curve in the high-SNR region. If the average BER expression, P_e, as a function of SNR per bit (or β) is available, we can find the coding gain G_c and diversity gain G_d of any systems using the definition,

$$\lim_{\beta \to \infty} P_e = (G_c \beta)^{-G_d} \Rightarrow \lim_{\beta \to \infty} P_e \beta^{G_d} = (G_c)^{-G_d}. \tag{I.1}$$

By replacing β with $\frac{1}{x}$ we can apply the Taylor expansion of the average BER expression around $x = 0$ (instead of working with the expression around $\beta \to \infty$) and find the diversity gain and consequently the coding gain using the fact that,

$$\lim_{x \to 0} P_e x^{-d} = \begin{cases} 0 & \text{if } d < G_d \\ {G_c}^{-G_d} & \text{if } d = G_d \\ \infty & \text{if } d > G_d \end{cases} \tag{I.2}$$