Die Casting Design. A Parametric Approach

by

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Abstract

This work presents an enhancement of the current gating system design methodology based on a better understanding of the physics of the die casting process and the availability of simulation software. Accurate values of critical parameters involved in the die casting design process such as filling time and freezing time are calculated to allow more knowledgeable decision making during the design stage. The calculation of the pressure required by the die to produce a given casting was improved by considering the influence of the static pressure from the air exhaust system. The die casting machine capabilities and die requirements are matched using the Machine Performance Envelope on a P–Q^2 diagram. Two different design scenarios were proposed. A design environment with scripting capabilities was implemented to provide a flexible user-driven design process. An evolving design scenario is presented to illustrate the use of the design environment.
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Nomenclature

a   Machine Performance Envelope constant
A   area of shot sleeve / cross sectional area of the duct
A_h area of the shot cylinder
A_g area of the gate
c   speed of sound
C   perimeter of the air exhaust line
C_t composite discharge coefficient
c_p specific heat at constant pressure
D   diameter of the shot sleeve
D_h hydraulic diameter of an air exhaust line
f   friction factor / body forces
F   scalar force
F   force vector
f_p filling percent of the shot sleeve
f_s fraction of solid per unit volume
E_i energy of the fluid at the conduit inlet
E_o energy of the fluid at the conduit outlet
\( h \) specific enthalpy

\( H_t \) total energy losses

\( k \) air polytropic compression constant

\( K_c \) slope of the die line on \( P-Q^2 \) diagram

\( K_t \) total hydraulic system coefficient

\( K_v \) hydraulic system coefficient

\( L \) length of an air exhaust line / the latent heat of diffusion

\( L_1 \) fictitious length of an air exhaust (unchoked conditions)

\( L_2 \) critical (total) length of an air exhaust (unchoked conditions)

\( L_s \) stroke length of the shot sleeve

\( \mathcal{M} \) Mach number

\( \tilde{\mathcal{M}} \) air molecular weight

\( \dot{m} \) mass flow rate

\( P \) pressure on the plunger face

\( p \) pressure of the air in the air exhaust line

\( P_1 \) inlet static pressure of an air exhaust line

\( P_2 \) outlet static pressure of an air exhaust line

\( P_{01} \) stagnation pressure of air in the die cavity

\( x_i \)
\( \text{Pa} \) atmospheric pressure
\( P_{\text{acc}} \) accumulator pressure
\( P_d \) pressure required by the die for a given design
\( P_h \) pressure in the shot cylinder
\( P_i \) pressure at the conduit inlet
\( P_m \) maximum pressure the machine can provide for a given design
\( P_o \) pressure at the conduit outlet
\( Q \) flow rate
\( Q_{\text{max}} \) maximum flow rate / dry shot flow rate
\( Q_{\text{fast}} \) fast shot flow rate
\( Q_{\text{slow}} \) fast shot flow rate
\( R \) gas constant
\( \tilde{R} \) universal gas constant
\( R_h \) hydraulic radius
\( s \) entropy per unit mass
\( S_{\text{dc}} \) knowledge of die casting process space
\( S_{\text{die}} \) knowledge of the die space
\( t_f \) filling time
\( t_{\text{fast}} \) fast shot time
\( t_{\text{fill}} \) filling time
\( t_{\text{freezing}} \) freezing time
\( t_{\text{slow}} \) slow shot time
\( T \) temperature
\( T_1 \) inlet temperature at the air exhaust line
\( T_2 \) outlet temperature at the air exhaust line
\( T_{01} \) stagnation temperature in the die cavity
\( T_a \) atmospheric temperature
\( u \) velocity
\( v \) shot sleeve velocity (velocity of the plunger inside the shot sleeve) / velocity of the air in the air exhaust line
\( \vec{v} \) velocity vector
\( v_{\text{dry}} \) dry shot velocity
\( v_g \) velocity of the liquid metal at the cavity gate
\( v_i \) velocity at the conduit inlet
\( v_o \) velocity at the conduit outlet
\( \text{Vol} \) total volume of the metal through the gate
\( V_c \) volume of casting
\( V_o \) volume of the overflows
\( V_T \)  total volume of liquid metal
\( \alpha \)  angle between force and velocity vectors
\( \kappa \)  thermal conductivity
\( \mu \)  dynamic viscosity
\( \Psi \)  volume fraction of liquid function (VOF)
\( \rho \)  density of the fluid
\( \rho_1 \)  density of air at the air exhaust inlet
\( \rho_{01} \)  stagnation density of air in the die cavity
\( || \)  vector modulus

\( (*) \)  critical conditions where the Mach number is equal to 1
List of Acronyms

- A -

accumulator pressure  Pressure provided by the hydraulic accumulator of a die casting machine.

air exhaust line (AEL)  Constant cross section rectangular duct. typically with 1 or 2 mm in height, 25 mm width and 200 mm length.

air venting  Extraction of air from the die cavity.

air exhaust system  System used to evacuate air from the die cavity.

ADCI  American Die Casting Institute.

- B -

biscuit  Excess metal that remains in the shot sleeve between the plunger and the parting surface of the ejector die half.

- C -

cold shut  It occurs when the metal in the die cavity freezes before the die cavity is completely filled.
cooling line Constant cross section circular pipe used to remove the heat from the casting by circulating a coolant fluid such as water or oil.

cooling system System used to remove the heat from the casting alloy. It is composed of several cooling lines.

- D -

die casting cycle Time required to open, close and fill the die cavity

die casting machine injection system System used to push the molten metal into the die cavity

Die Design Line (DDL) Line which expresses the pressure requirements for the die at any flow rate considering the viscous pressure drop and back pressure from the air in the die cavity.

Die Line (DL) Line which expresses the viscous pressure for flow into a die at any flow rate.

die open time Time that the die remains open during the die casting cycle

die closed time Time that the die remains closed during the die casting cycle
**dry flow rate** Flow rate when the plunger moves at dry shot velocity

**dry shot velocity** State at which the all energy from the injection system is used to move the plunger at zero pressure.

- F -

**fast shot** Shot used to fill the die cavity

**feeding of shrinkage** Metal pressure intensification at the end of the injection of the molten metal into the die cavity.

**filling time** Time required to fill the die cavity.

**flexibility** It considers the changes that could be made in the filling time and gate velocity variables using only soft variables.

**fluid flow system** System which considers the metal injection, air venting and feeding of shrinkage.

**freezing time** It is usually defined to be the time at which the first molten metal entering to the die cavity reaches 4% of the fraction solid.
- G -

gate Slit like channel connected to the runners used to fill the die cavity. Also called ingate.

gate depth Gate height.

gate thickness Gate height.

gate velocity Average velocity at the gate to the cavity.

gating system A series of passages through which the molten metal flows into the die cavity. It is composed of the runner(s), gate(s) and biscuit.

gooseneck Elongated “S” shaped passage which connects the molten metal reservoir to the nozzle in a hot chamber die casting machine.

- H -

hard variables Variable set that requires changes in the die and/or machine usually a disassembly in order to be varied. The set includes parameters such as plunger diameter, gate area and air exhaust location.

xviii
injection system  System used to push the molten metal into the die cavity.

   It is a hydraulic system powered by a hydraulic pump driven by an
   electric motor.

- M -

Machine Line (ML) Line which expresses the die casting machine capa-
   bilities for any flow rate between zero and the dry shot flow rate.

Machine Performance Envelope (MPE)  Envelope curve which expresses
   the die casting machine capabilities for any flow rate. Every tangent
   to the MPE is a Machine Line.

- N -

nozzle  Passage used to connect the gooseneck and the die cavity in a hot
   chamber die casting machine.

- O -

Operational Window (OW)  Window obtained as a result of setting the
   boundary values for the filling time and gate velocity.
overflows Chambers outside of the die cavity used to facilitate the evacuation of undesired material from the die cavity.

- P -

**P–Q² Diagram** volumetric flow rate squared ($Q^2$) of the molten metal vs. pressure on the plunger face ($P$) diagram. It is used to represent both the die requirements (DL) and the die machine capabilities (ML).

**P–V Diagram** Velocity of the molten metal ($V$) vs. pressure on the plunger face ($P$) diagram used to represent both the die requirements (DL) and the die casting machine capabilities (ML).

**Plunger** Actuator (piston) which pushes the molten metal into the die cavity at a desired flow rate.

**Pouring hole** hole located on the shot sleeve used to pour the molten metal inside the shot sleeve at a given pouring rate.

- R -

**Runners** Channels, usually with trapezoidal cross section used to conduct the molten metal from sprue hole or shot sleeve to the die cavity.
shot sleeve Constant cross sectional circular duct where the molten metal is poured to be injected into the die cavity.

slow shot Shot used to bring the molten metal up to the die cavity gate.

soft variables The set of variables that can be easily changed during the die casting process operation. This set is composed of variables such as accumulator pressure, metal pressure, pouring temperature, die temperature and cycle time.

sprue Metal solidified in the sprue hole.

sprue hole Tapered hole through the thickness of the cover die used by the molten metal to reach the die cavity.

stroke length Portion of the shot sleeve length used to perform the slow and fast shots.
- V -

vent  see air exhaust line.

venting  see air exhaust system.
Chapter 1

Introduction

1.1 Die Casting Processes

Die casting can be described as a process by which hydraulic energy from an injection system of a die casting machine is applied to molten metal to convey kinetic energy to the metal to achieve a fast filling of the die cavity.

There are several die casting systems in use. Primarily the fluid flow, air removal from the cavity, heat loss during injection and reactivity between the molten metal and the hydraulic system distinguish each of these systems. Although they have distinctive characteristics, these die casting methods have similar mechanical design of the die, thermal control and actuation.
Figure 1.1: Schematic showing the principal components of a hot chamber die casting machine after Sully [19].

The main two die casting processes are the hot chamber process and cold chamber process [19].

The hot chamber process (Figure 1.1) is employed for materials such as zinc, lead, tin and magnesium which have lower melting points. In this case, several system components are continuously in direct contact with the molten metal so that the exposure of the molten metal to phenomena like turbulence, oxidizing air, and the cooling of the molten metal during its
injection is minimized. The extensive contact between the molten metal and the components of the system represents a major drawback of this method for materials with higher melting points since the components of the system are severely affected.

To reduce the above mentioned difficulties, the cold chamber system developed (Figure 1.2). In this process the reservoir of molten metal and the hydraulic piston are separated during the majority of the cycle time. The metal has to be metered independently, poured into the shot sleeve and injected into the die. This process lasts only a few milliseconds and the exposure time of the hydraulic piston to the molten metal is as short as possible. This allows die castings of material of higher melting point such as aluminum and copper to be produced.¹

High speed involved in the die casting process allows a high level of production, on the order of one to 100 parts per minute per cavity depending on size and shape of the casting [19]. The repeatability, consistency, surface finish and appearance of the die casting process are good and tolerance is moderate [13].

¹Some ferrous alloy castings have been produced by this method.
Figure 1.2: Schematic showing the principal components of a cold chamber die casting machine after Sully [19].

Die casting becomes a cost effective process when a high production volume is maintained. The manufacturing cell cost is high. The construction of a die for the given part is an expensive and long term task and setting up the die casting machine to produce quality castings may require some time [13].

1.2 The Gating Fluid Flow System

Die casting demands a close interaction between the process design and the product design. The process control to accomplish consistent high quality of the final product involves parameters such as the cycle time, the fluid flow, heat flow and dimensional stability. These parameters are decided at
the design stage or when the die is mounted in the machine, but they are required to match the final design.

Metal injection, air venting and feeding of shrinkage are the three primary considerations of the fluid flow in the die casting process [19]. The injection of the molten metal is characterized by its high velocity and hence short fill time of the cavity is achieved. The success of the process relies on properties such as temperature, composition, gas content and the amount of suspended solid.

The injection chamber is composed of the shot sleeve in the cold chamber process and the gooseneck and the nozzle in the hot chamber process. To avoid turbulence, eddies, wave formation and air entrapment the shot sleeve is filled by slow movement of the piston. The molten metal is then accelerated to the required injection velocity, independently of the die casting system in use.

After the die cavity is almost completely filled with molten metal, extra pressure is applied on the molten metal to facilitate filling the cavity with metal.

A smooth transition from the injection chamber to the die cavity is arranged by the runners and sprues. They also provide high cooling heat flow
after the injection of the cavity is achieved. The molten metal travels from the injection chamber through the runners to the cavity periphery where the gate to the cavity is located. The design of the runners influences heat loss, turbulence and die erosion.

The last component of the liquid metal fluid flow system are the overflows which hold the molten metal that has to be removed from the die cavity and ease the casting extraction and manipulation.

The air exhaust and the cooling systems are two other components of the die casting process which are not part of the liquid metal fluid flow system. They influence the quality of the casting.

The air exhaust system allows for extraction of the entrapped air inside the fluid flow system after it is sealed. When the plunger passes over the pouring hole in the cold chamber system. The effective removal of the air permits better metal properties to be accomplished and the possibility of heat-treatment of castings.

Several methods can be used to evacuate the entrapped air from the fluid flow system. The most common one is the location of air exhaust lines or vents in specific places based on the experience of the designer and a trial and error procedure when the die is mounted on the die casting machine. The air
can also be extracted by a vacuum system before metal filling. The application of the pore-free process eliminates the entrapped air by the reaction of the molten aluminum with oxygen.

Heat removal from the die cavity through cooling lines is paramount as it permits the solidification and cooling of the casting. In this case, the die cavity performs a heat exchanger role. A large gradient in the casting causes shape distortion, hot tears and cracks. The effectiveness of the heat removal depends on factors such as the cooling line location, coolant flow rate, temperature and pressure.

In cases when the die geometry substantially affects the heat loss during the filling, hot oil can be used as a coolant fluid. The heat removal system can preheat the die before the filling. This approach contributes to a longer die life.

Once solidified and cooled, the casting has to be removed from the cavity. Ejection of the cast uses ejector pins. The location of the such pins is influenced by the casting geometry. After ejection the preparation for the next cycle starts. External water sprays are often applied to decrease the temperature of the die surface. Residual water and other remains of the previous casting are removed.
This step is followed by the application of lubricant which creates an insulating layer between the die surface and the casting. The shot cylinder is also lubricated. Finally the die closes and it is ready for the next cycle.

1.3 Design of the Gating System

The design of the gating system is an important factor to produce high quality castings. It should consider factors such as part shape, internal quality, surface quality, mechanical properties, die material, temperature, erosion, and venting, as well as metal temperature, fluidity, heat content and microstructure [19].

It is important to realize that despite research efforts, the design of a gating system still has an experience component that cannot be neglected, especially in the gate location and local angle of entry. These two factors are directly related to the shape of the part to be cast and secondary operations such as trimming.

Sully [19] reports the American Die Casting Institute (ADCI) Nomograph as one of the pioneer analytical methods for gate system designs. Based on this nomograph, the geometric relationships for the bulk flow design can be
established. The selection of the fill time for a given casting relies on the experiments and experience of the designer. A limited number of available plunger diameters and the percent of filling of the shot sleeve in the cold chamber process constrains the required flow rate for a given casting.

The next step in the development of analytical tools was the introduction of the $P-Q^2$ diagram by CSIRO Australia. The $P-Q^2$ diagram shows that the pressure on the molten metal has a quadratic relationship to the flow rate. It also shows how the pressure available from the machine is not constant and that it depends on the flow rate. The pressure demanded by the gating system can be simultaneously shown on this diagram too. CSIRO also proposed the filling pattern and tapered runners.

The introduction of a complete design methodology by Herman [12] was another important effort in providing an analytical formulation of the gating system. The methodology furnishes the designer with a mix of analytical and experience-based approaches which often guarantee the production of good castings. Since its introduction the understanding of several aspects of the methodology have been enhanced and therefore a more knowledgeable design can be achieved.

Karni [13] introduced the Machine Performance Envelope approach as a
generalization of the die casting machine capabilities. Karni also provided tools to increase flexibility and optimize the design to allow an easier search for an optimal operating point.

Individual aspects of the die casting process such as filling and solidification have been studied [3], [6] and [20]. Tools for simulating the casting filling and solidification were introduced. The availability and generalization were limited by the powers of the computers.

More recently the air exhaust system has been studied [1]. In this work a complete analytical model for modeling the venting system was developed. As a result, it is now possible to optimize the area of the air exhaust lines in order to satisfy the air evacuation requirements.

1.4 Software for Designing Gating Systems

There are several software packages that implement gating design methodologies. "METALFLOW" from Moldflow, Australia Pty. introduced the fill pattern and the tapered runners. The fill patterns schematically describe the metal flow from the gate into the die cavity. The user is responsible for selecting the desired filling pattern for the casting and uniform filling condi-
tions are obtained by the system. Also, the flow length and the venting are analyzed. The final result combines considerations of the filling pattern and the setup point of the die casting machine to determine the fraction of solid at the end of the filling.

"Runner Design", "Metal Flow Predictor” and "Feed Design" from the American Die Casting Institute are other software packages used for calculating the gate system [7].

The "Flow Predictor" module analyzes the effect of the plunger size, hydraulic pressure, shot speed control and gate size on the filling conditions. Based on the information provided by this module the "Runner Design” module can be used.

The “Runner Design” is used to calculate the runner system to the exact size since the flow rate is known. The concept of unit construction and standardized parts is applied in this module.

Finally, in the “Feed Design” module the calculation of the feeding system is performed. It guarantees that the filling conditions will be met and the predictability of the performance.

The Computer Integrated Die Casting (CIDC) is based on solid modeling techniques [13]. It integrates different software tools into a design environ-
ment, avoids reiteration of the geometry definition and it can be used for structuring the design process. The material, tools, process, product and engineering analysis are included in this software. The die casting process is modeled by a finite element implementation which simulates the fluid flow of filling the cavity and heat transfer during cooling. The flow segmentation and the process variables are selected by the user based on experience. It does not consider the influence of the air exhaust system in the process.

Rao [18] described a software package for designing the gate system. It uses the binary search method for determining the filling time and gate velocity. The system also permits the design of different runner and gate types.

An updated version of the “METALFLOW” software has been recently introduced [14]. There are two modules, “Castflow” and “Casttherm”. Castflow is a fill pattern based system that provides the runner geometry for flow analysis and a 3D wire-frame that can be used for tool path generation. Casttherm performs the thermal analysis of the cavity filling.

The present work covers improvements in the gating design methodology based on recent advances in the understanding and modeling of the die casting process.

Accurate values obtained from the simulation of the filling of the die cav-
ity, the thermal analysis of the die and the casting and the advective energy analysis are used in order to make the design process more knowledgeable.

A new software module for doing parametric die casting design has been developed and a flexible die casting design environment is implemented.
Chapter 2

The P–Q² Diagram

2.1 The P–Q² Diagram

The P–Q² diagram is an important tool for the die casting design process. It was originally developed by CSIRO in Australia in 1977 [22], although its basis can be found in the P–V² diagram introduced two years earlier [23].

The theoretical basis of the P–Q² diagram is a statement of the conservation of energy for constant density, steady incompressible flow [21]. It states that energy of the fluid at a conduit inlet has to be equal to energy of the fluid at its outlet plus losses and it can be expressed as:
\[ E_i = E_o + H_t \]  \hspace{1cm} (2.1)

where:

- \( E_i \): energy of the fluid at the conduit inlet;
- \( E_o \): energy of the fluid at the conduit outlet;
- \( H_t \): total energy losses.

Equation (2.1) expands to a relation which uses parameters that can be measured and are usually available to die casters, such as velocity, pressure and liquid metal properties. It takes the following form [21, 22]:

\[
\frac{P_i}{\rho} + \frac{1}{2} v_i^2 = \frac{P_o}{\rho} + \frac{1}{2} v_o^2 + H_t
\]  \hspace{1cm} (2.2)

where:

- \( P_i \): pressure at the conduit inlet;
- \( P_o \): pressure at the conduit outlet;
- \( v_i \): velocity at the conduit inlet;
- \( v_o \): velocity at the conduit outlet;
- \( \rho \): density.
Figure 2.1: Schematic showing the shot sleeve, runner, gate die cavity and vent after Bar-Meir et al. [1].

Figure 2.1 shows the fluid flow system used in a die casting process to inject the molten metal poured into the shot sleeve into the die cavity.

The energy losses while the molten metal travels from the shot sleeve to the gate of the die cavity are mainly due to friction, geometry changes and the jet flow at the gate.

Due to the non-homogeneous nature of the molten metal, its constituents do not travel at the exact same rate or direction through the shot sleeve and runner-gate system, causing turbulence, collisions and eddies in flow stream.
The high velocities used to inject molten metal in the die casting process and conditions such as surface roughness, abrupt changes in conduit size and sharp bends also contribute to such a behaviour.

Assuming that the pressure in the die cavity is almost zero and considering the influence of the different components of the fluid flow system, equation (2.2) can be expressed as:

\[ P = C_t v_g^2 \]  \hspace{1cm} (2.3)

where:

- \( P \): pressure on the plunger face;
- \( v_g \): velocity of the liquid metal at the cavity gate;
- \( C_t \): composite discharge coefficient.

Equation 2.3 is used to compute the pressure required at the plunger face to achieve the desired liquid metal velocity at the point of discharge, which in this case is considered as the gate of the cavity.

The value of the \( C_t \) coefficient can be calculated by measuring the injection speed of the die casting machine. Die casting practice shows that the value of the composite discharge coefficient and the construction of the P–Q²
or P–V² diagrams are often based on the measured die casting machine performance rather than the machine specification itself [12]. In other words, the Cᵢ coefficient not only depends on the die casting machine specification but also on other components of the die casting process such as the runner, gate and air exhaust systems. Equation (2.3) applies to both cold and hot chamber die casting processes.

### 2.2 Die Line

A relationship between the flow rate and the pressure (P) can be obtained if the velocity at the gate of the cavity term (vₙ) is expressed as a function of the flow rate (Q) and the gate area (Aₙ) as follows:

\[
    vₙ = \frac{Q}{Aₙ}\quad (2.4)
\]

Then, substituting equation (2.4) into (2.3), yields:

\[
    P = \frac{C_i}{A^2} Q^2\quad (2.5)
\]

This shows that the pressure on the plunger face has a quadratic dependency on the flow rate, thus to increase the flow rate, the pressure P has to
increase. This is usually achieved by increasing the hydraulic pressure. This behaviour is shown in Figure 2.2

The high injection velocities used in the die casting process suggest a turbulent behaviour of the flow. Considering that gate velocity usually varies between 30 to 60 m/s (see Section 3 for details), the Reynold’s number takes values between $30 \times 10^3$ to $120 \times 10^3$ (fully turbulent flow) for a gate height of 2 mm and kinematic viscosity of $10^{-6}$ at 700 °C for Al380 alloy.

A similar analysis taking into account plunger velocities of the die casting machines between 2 and 4 m/s in the fast shot yields values of the Reynold’s number ranging from $3.75 \times 10^5$ to $1.0 \times 10^6$ for plunger diameters between 150 to 300 mm. This too suggests the flow behaviour is fully turbulent.

For sake of simplicity, the plot of the flow rate ($Q$) vs pressure on the plunger face ($P$) in Figure 2.3 is shown as a straight line with positive slope using a quadratic flow rate scale instead of a linear one.

The composite discharge coefficient is considered to be a measure of the efficiency of the die casting process in general but more specifically it measures the efficiency of the runner-gate system.

Figure 2.4 displays the influence of the composite discharge coefficient on the pressure that has to be supplied by the injection system. An improvement
Figure 2.2: P–Q diagram showing Die Line for various flow rates. The flow rate axis is constructed using a scale linear in Q.

Figure 2.3: P–Q² diagram showing Die Line for various flow rates. The flow rate axis is constructed using a scale linear in Q².
Figure 2.4: P–Q² diagram showing the effect of the discharge coefficient (Cₜ) on the Die Line. Gate area Aᵣ = 0.0151 m².

In the discharge coefficient value can considerably increase the available flow rate or decrease the pressure required from the injection system for a fixed flow rate. This has a strong influence in the casting quality (see Section 3).

The composite discharge coefficient is often assumed in practice. Usually Cₑ ≈ 0.5 - 0.6 for zinc [22] and Cₑ ≈ 0.5 for aluminum [2].

2.3 The Machine Line

Since the P–Q² diagram shows the relationship between the metal pressure on the plunger face and the flow rate, it is possible to display on the same
diagram, how the pressure demanded by the die relates to the pressure available from the die casting machine. This approach provides an understanding of the real capacity of a machine to successfully produce a given casting [23].

There are two well defined extremes in the behaviour of the die casting machine injection system. One, when all the supplied power is merely used to move the plunger without any resistance from the molten metal. and the other when the maximum pressure of the accumulator is applied with zero flow rate.

When the shot sleeve does not contain any molten metal, there will be no fluid resistance that the plunger has to overcome to move. In this case the energy provided by the injection system is used for moving the plunger and not for pushing any molten metal and it will increase up to a point where there is no more energy available from the injection system. At this point, the maximum possible value of the plunger velocity has been reached and all the involved losses are caused by the hydraulic circuit. This defines the dry shot velocity and there is no energy available to push the liquid metal [12]. It is also considered to be a measure of shot circuit fluid flow efficiency [23].

The above explanation shows that to increase the dry shot velocity the resistance of the shot end circuit has to be reduced. Mathews and Krett
[13] and Cocks and Wall [8] have proved that the dry shot velocity was considerably increased when a more efficient valve was installed in a shot circuit.

The second of the aforementioned extremes can be described as an static stage where the maximum allowed pressure from the accumulator is available but no movement of the plunger is involved and therefore there is no fluid flow.

A mathematical formulation of the Machine Line (ML) can be obtained if the equation 2.1 is applied to the die casting machine injection system, shown schematically in Figure 2.5.

The major loss sources are due to the existing fitting and to a speed control valve of a second phase.
Considering that the input pressure remains constant and it is equal to the pressure supplied by the accumulator, the shot sleeve is full of molten metal the Machine Line can be expressed as follows:

\[ P_h = P_{acc} - K_v v^2 \]  

(2.6)

where:

- \( P_h \): pressure in the shot cylinder;
- \( P_{acc} \): accumulator pressure;
- \( K_v \): hydraulic system coefficient;
- \( v \): shot sleeve velocity.

The pressure on the plunger face \( P \) in the shot sleeve is proportional to the pressure \( P_h \) in the shot cylinder. The equilibrium of forces yields the relationship

\[ PA = P_h A_h \implies P = P_h \frac{A_h}{A} \]  

(2.7)

where:

- \( P \): pressure in the shot sleeve on the plunger face:
Figure 2.6: P–Q diagram showing the Machine Line. The flow rate axis is constructed using a scale linear in Q.

A: area of the shot sleeve;

\( A_h \): area of the shot cylinder.

Expressing \( v \) in terms of the flow rate \( (v = Q/A) \), permits one to express (2.6) in the form shown in equation (2.8) as originally proposed by Karni [13].

\[
P = P_{acc} \frac{A_h}{A} - K_t Q^2
\]  

(2.8)

where:

\( K_t = K_v/A^2 \).

Figure 2.6 represents a Machine Line using a linear in Q scale. This
Figure 2.7: P–Q$^2$ diagram showing the Machine Line. The flow rate axis is constructed using a scale linear in Q$^2$.

The graph shows how the die casting machine capabilities varies depending on the flow rate. A larger flow rate demands a larger fraction of the power available from the machine to be used to move the plunger at the desired velocity. This decreases the pressure available on the plunger face to produce the casting.

As in the case of the Die Line, the representation of the equation (2.8) becomes a straight line if a quadratic scale is used on the P–Q$^2$ diagram as shown in 2.7. In that case, it represents a straight line with a negative slope in which the intercepts are the extreme points previously analyzed. The evaluation of equation (2.8) for such points gives:
for $Q = 0 \implies P = P_{acc} \frac{dA}{A}$

for $Q = Q_{max} \implies P = 0$, since $P_{acc} \frac{dA}{A} = K_t Q_{max}$ as defined above.

It should be noted that the pressure available from the machine is a function of the flow rate which at the same time depends on the plunger velocity. Furthermore, the movement of the plunger is affected by the static pressure on the molten metal from the air exhaust lines. Details of the air exhaust model utilized are given in Appendix A.

Research work at CSIRO in Australia showed that the representation of the pressure on the plunger face or metal pressure versus metal flow rate squared is one of the most suitable approaches to understand and represent the capability of a die casting machine injection system [23].

The Machine Line on a $P-Q^2$ diagram is affected by factors such as: the accumulator pressure, the shot speed control valve aperture and the diameter of the plunger.

Equation (2.8) shows that when the accumulator pressure increases, the pressure available on the plunger face also increases. Since there is more power from the injection system to be used, the dry shot velocity (dry shot flow rate) is higher. This behaviour is shown in Figure 2.8 where the new
Figure 2.8: P–Q² diagram showing the effect of increasing the accumulator pressure on the Machine Line.

On the other hand, if a larger value of the plunger diameter is used, the area of the shot sleeve also becomes larger, increasing the dry shot flow rate. As a result of these changes the first term in equation (2.8) decreases. The second term increases, producing an overall pressure reduction on the plunger face. This analysis has limited validity since the changes in mass and inertial forces required to move a larger plunger are not considered.

Figure 2.9 shows a comparison of two Machine Lines considering different values of the plunger diameter. The Machine Line with the largest plunger diameter has a larger dry flow rate but a smaller pressure on the plunger face.
can be achieved. On the other hand, the Machine Line for the small diameter could reach a higher pressure value and the dry flow rate is reduced.

Finally, the shot speed control valve affects the resistance of the fluid flow in the injection system. If the valve is wide open, the losses are at a minimum and the dry shot velocity reaches its maximum value. Every turn of the valve increases the losses and the decreases the dry shot velocity.

It should be also noted that despite the influence of the shot sleeve control valve in the dry shot velocity, the pressure available from the accumulator remains unchanged.

Figure 2.10 shows a reduction of the dry flow rate value as a result of a
higher losses in the injection system. The smaller the aperture of the second phase valve, the larger the losses. This losses affect the maximum value that the dry shot velocity of the machine can reach.

Alternatively, the behaviour of the die casting machine injection system can be described as a function of the actual shot velocity to dry shot velocity ratio given by equation (2.9). This expression was originally proposed and experimentally tested by CSIRO [23].

\[ P = P_{acc} \frac{A_h}{A} \left(1 - \frac{v^2}{v_{dry}^2}\right) \]  

(2.9)

where:
\( v_{dry} \): dry shot velocity;

\( v \): velocity of the plunger.

In the above expression, all the losses already described are accounted for in the \( v \) term.

Cocks and Wall [8] outlined the ideal performance characteristic for a die casting machine. They suggest that the available pressure should be between 8 and 26 MPa. This range guarantees a proper gate velocity. Next, in order to have a better control of the filling time, the effect of the gate area on the gate velocity has to be diminished. This can be achieved using Machine Lines with the flattest possible slope. They also recommended the use of the highest possible value for the dry shot velocity.

### 2.4 The Machine Performance Envelope

The Machine Line described in Section 2.3 is defined for a specific value of the plunger diameter. This approach can be generalized if different Machine Lines are obtained for the same die casting machine.

The Machine Performance Envelope (MPE) is created if all possible Machine Lines are plotted on \( P-Q^2 \) diagram. This includes such hypothetical
cases as zero and infinite diameter values. This infinite set of lines defines an envelope curve which has one tangent point for each Machine Line.

Assuming that the power from the injection system remains almost constant, it can be expressed as follows:

\[
Power = \bar{F} \cdot \bar{v} = |\bar{F}| |\bar{v}| \cos(\alpha) \approx \text{const} \quad (2.10)
\]

where:
\( \bar{F} \): force;
\( \bar{v} \): velocity in the shot sleeve;
\( \alpha \): angle between the force and velocity vectors;
\(| | \): vector modulus.

Since the force \( \bar{F} \) and the velocity \( \bar{v} \) vectors are both in the same direction in the shot sleeve, \( \alpha \) becomes zero. The power can be expressed as:

\[
Power = Fv \quad (2.11)
\]

Substituting \( v = Q/A \), (\( A \) is the area of the shot sleeve) and \( P = F/A \), the equation (2.11) can be restated as:
This expression represents a hyperbola which is tangent to all Machine Lines. It also shows that the MPE depends on the accumulator pressure (\(P_{\text{acc}}\)), the shot piston area (\(A_h\)) and the dry shot velocity.

It is important to note in the above analysis that the Machine Performance Envelope is independent of any plunger diameter or die design. It only relates to the die casting machine, hence it is possible to consider that the MPE varies continuously in \((Q^2, P)\) space [11].

A relationship between the pressure and the flow rate can be derived. Then equation (2.12) takes the following form:

\[
Power = P_{\text{acc}} \frac{A_h}{A} Q = P_{\text{acc}} A_h v \approx \text{const}
\]

(2.12)

where:

\(P\): pressure on the plunger face (\(P = P_{\text{acc}}(A_h/A)\)) and
\(a\): Machine Performance Envelope constant.

It can be shown (Karni [13]) that
Figure 2.11: $P-Q^2$ diagram showing the Machine Performance Envelope.

![Diagram showing $P-Q^2$ relationship](image)

$$a = \frac{2\sqrt{3}}{9} \cdot A_h \cdot \frac{(P_{acc})^{3/2}}{K_t^{1/2}} = \frac{2\sqrt{3}}{9} \cdot A_h \cdot P_{acc} \cdot v_{dry}$$ (2.14)

This constant represents the energy supplied by the accumulator which is released to the molten metal through the injection system in the injection phase. It has dimensions of power.

A Machine Performance Envelope curve plotted on $P-Q^2$ diagram is shown in Figure 2.11. It shows how the pressure on the plunger face decreases as the flow rate increases.

The new formulation of the Machine Line and the intersection point between the Machine Line and the Die Line, considering the MPE constant $a$ is:

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Figure 2.12: Intersection point of the Machine Performance Envelope and Machine Line after Karni [13].

\[ P = P_0 \left( 1 - \frac{4}{27} \frac{P_0^2 Q^2}{a^2} \right) \]  \hspace{1cm} (2.15)

and,

\[ Q_1^2 = \frac{P_0}{\frac{C_2}{a^2} + \frac{4}{27} \frac{P_0^2}{a^2}} \]  \hspace{1cm} (2.16)

respectively (see [13] for details).

Figure 2.12 shows the tangent point between a Machine Line and the Machine Performance Envelope of a die casting machine. The point \( P_1 \) determines the maximum pressure that a die casting machine can provide for \( Q_1 \) flow rate.
The Machine Performance Envelope has the advantage that it allows an independent formulation of the die and die casting machine influences. This feature is useful in die casting practice where the same die casting machine is utilized to perform several castings.
Chapter 3

Operational Window

The establishment of boundaries on relevant variables in a given process defines an Operational Window (OW). It usually guarantees an appropriate (known) behaviour of the process within its limits, and the violation of such boundaries could cause unexpected results.

The Operational Window for the die casting process is set according to the filling time, gate velocity and final metal pressure [13]. These variables are considered to have a strong impact on the quality and manufacturability of the final product (see Section 3.1 for details) and their values are defined or estimated using the geometry and specifications. The use of experimental data or software tools such as heat transfer, solidification and flow simulations

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are often needed in order to obtain their actual values.

The pressure axes is used to describe the gate velocity. Using the relationship between the pressure on the plunger face and the gate velocity that was obtained in the equation (2.3), the gate velocity can be expressed as:

\[ v_g = \sqrt{P_{acc} \frac{A_h}{A}} \frac{1}{C_t} \]  \hspace{1cm} (3.1)

The flow rate axes can be used to characterize the filling time. This transformation is accomplished by means of the expression:

\[ Q = Av = \frac{Vol}{t_f} \]  \hspace{1cm} (3.2)

where:

Vol: total volume of the molten metal through the gate;

\( t_f \): filling time.

or in terms of filling time:

\[ t_f = \frac{Vol}{Q} \]  \hspace{1cm} (3.3)

It is important to mention that in this case, the filling time scale has its
largest value at the origin of the diagram since for a larger flow rate a shorter filling time is required.¹

3.1 Constraints on Operational Window

The Operational Window boundaries and their effects on the production of castings are shown in Figures 3.1 and 3.2 respectively.

Zone A is the limit of the maximum filling time. Its value has to be smaller than the freezing time. Violating this limit causes poor surface quality in casting, cold shuts and incomplete filling because the molten metal will freeze before the cavity has been filled [8].

Zone B is the minimum allowed gate velocity (30 m/s). This value is required to atomize the flow, i.e., the molten metal comes into the cavity as a spray. Gate velocity values smaller than this limit cause porous casting [8].

Zone C is the minimum filling time. This value is obtained from the venting condition of the die, i.e., the air inside the die cavity should be pushed out of the cavity by the molten metal entering the die cavity. This

¹In this analysis the volume of the injected molten metal is considered constant.
value is limited by the maximum pressure available from the plunger.

Zone D is the limit for the maximum gate velocity. Its value has to be strictly smaller than 60 m/s. A higher gate velocity value causes die soldering and erosion, decreasing the die life [8]. The literature is not clear about how the die soldering and erosion occur.

### 3.2 Flexibility

The Operational Window offers a starting point for an optimization since it formally guarantees casting of good quality.

The search for a global optimum of the die casting process variables is not feasible at the current level of understanding. Such a formulation should
try to minimize costs or to maximize productivity, considering the whole set of manufacturing factors and their close interrelations. It also requires more experimental and modeling work.

An alternative to the global maximum approach is to try to find individual optimum values for each process variables. This is the strategy used here.

The limitation described above and the complexity of the die casting process do not allow one to determine the optimum values at the design stage. The search for an optimum operational point is achieved by providing flexibility around the operational point.

The die casting variables can be grouped into soft and hard variables according to the ease with which they can be changed.

Figure 3.2: Effect of Operational Window Boundaries on Castings.
Soft variables are those that can be easily changed during the die casting process operation. They include among others: accumulator pressure, metal pressure, pouring temperature, die temperature and cycle time.

Hard variables are defined as those that require changes in the die and/or machine, usually a disassembly. They include: plunger diameter, gate area, air exhaust location and size and cooling line positions.

The concept of flexibility is based on the changes that could be made in the filling time and gate velocity variables within the boundaries of the Operational Window. These changes must only consider new values of the soft variables.

3.3 Optimization of the Gate Area

According to equation (2.5), the slope of the Die Line depends on the gate area.

The extreme values for the gate area can be obtained from the Die Lines that cross the upper left corner and the lower right corner of the Operational Window as Figure 3.3 shows. The points $P_1$ and $P_2$ represent the minimum and maximum values respectively.
Figure 3.3: Extreme Values for the Gate Area.

For a given gate area in the above defined range the relationship between the filling time and the gate speed is established and then a specific Die Line can be plotted.

This Die Line intercepts the Operational Window in two points. They define a set of all possible filling time and gate velocity combinations. This set is constrained by the MPE and the boundaries of the OW. Therefore only points located within these limits are feasible.

The flexibility in the design can be graphically represented by the length of the DL within its constraints (Figure 3.4). The larger this length, the greater the flexibility around the setting point.

Alternatively, the constrained area under the DL can be considered as a
Figure 3.4: $P-Q^2$ and the area $A$ as a measure of flexibility after Karni [13].

representation of the flexibility. In that case, the greatest flexibility is offered by the greatest area.

It is worth noting that because the OW is graphically represented as a rectangle, the largest flexibility will be obtained when a Die Line intercepts the lower left corner and the upper right corner of the OW. Unfortunately such a Die Line seldom exits.

The other two optimal solutions occur when a Die Line intercepts either the lower left or the upper right corners of the OW.

A formulation has been proposed for calculating the optimum gate area value [13]. It defines the constrained area under the DL as the objective function. The optimization strategy tries to maximize its value.
Figure 3.5: Constraints of the Die Line after Karni [13].

It can be shown (see [13] for details) that the maximum of the objective function is obtained when the slope of the Machine Line $K_c$ is expressed as:

$$K_c = 27 \frac{P_1}{a^2}$$  \hspace{1cm} (3.4)

where:


The location of the MPE on the $P-Q^2$ diagram influences the value of the optimum to be used. There are two possible cases: (a) when the MPE is located above the OW and does not intercept it at any point; (b) when the MPE intercepts the OW.

In the former case, the optimum value for $K_c$ becomes $K_2$ (Figure 3.5). The Die Line which has $K_2$ slope crosses the upper right corner of the OW.
hence it allows a larger area when comparing to the Die Line with slope $K_1$.

The latter case when $K_b \leq K_c \leq K_d$, $K_c$ becomes:

$$K_c \leq \max(K_1, K_2)$$

then, the Die Line that provides the greatest area is used.

It should be mentioned that when calculating the gate area, that if the value of its height is fixed$^2$, then the calculation for different values of length could be done.

$^2$The height is about 1 or 2 mm in order to atomize the flow.
Chapter 4

Design Methodology

4.1 Current Approach

The design methodologies used by die casting companies are far from standardized. Based on well known general principles of the die casting process each company creates its own methodology which considers not only the general principles but also its own knowledge and experiences about the process. Each methodology also considers the die casting machines available to designers and other factors such as quality, cost and production goals. There could exist as many design methodologies as die casting companies.

Herman [12] proposed a methodology for designing the gating system
of castings. This methodology provides guidelines for the die casting design that are followed by most die casting companies to some extend. This methodology is summarized below.

1. Establish the metal flow or desired filling pattern.

2. Compute the filling time.

3. Select the gate velocity.

4. Establish the minimum and maximum gate depth.

5. Match the machine and die requirements.

6. Determine the final plunger size and slow shot velocity.

7. Calculate the final gate and runner dimensions.

The design process begins with the selection of the filling pattern, i.e., the desired flow of metal though the cavity. Although some guidelines and ideal filling patterns exist, the procedures are far from well understood or formalized in mathematics or a set of rules. Also, the suggested filling patterns are rarely achieved in real casting because the constraints involved in the development of the gate and runners.
This step includes the calculation of the total volume of metal through the gate. The volume of the cavity, overflows, air exhausts, flash and internal runner are considered. The decisions involved in this step mainly depend on the designer’s experience. The selection of the gate type is also considered here.

A detailed explanation about the filling patterns and guidelines can be found in [12]. This topic will not be discussed further in this work.

The second step of the methodology involves the calculation of an ideal filling time. The filling time is considered the time interval between when the molten metal starts to enter the cavity until the cavity and overflows are completely filled. It ends when the fast shot finishes.

The filling time is considered the most important variable that the runner-gate system and the machine operational parameter have to satisfy. Often it takes values between 0.01 and 0.06 seconds and is higher for larger castings. Also, shorter filling times provides good surface finish [8].

The expression for calculating the filling time depends on the temperature of the molten metal as it enters to the die, the minimum flow temperature of the metal, temperature of the die cavity surface just before the metal hits it, the percent of allowable solid fraction in the metal at the end of the filling.
casting thickness and an empirical derived constant. This calculation of the filling time is not accurate and it tends to underestimate the filling time for thick castings and castings with short flow distance.

The selection of the gate velocity is the third step in the procedure. The gate velocity depends on the runner size and machine capabilities. It is also constrained by the requirement to have atomized jet flow at the cavity gate. Furthermore, higher gate velocity values give good casting soundness [8]. The gate velocity and gate depth have a strong influence on the spray flow conditions.

The need for maintaining the gate velocity within the recommended boundaries of the Operational Window has been shown in Section 3.

On the other hand, the selection of gate velocity is constrained by economical considerations derived from the runner size. It includes the cost of re-melting the metal solidified in the runners. Larger gate velocity values allow runners of smaller cross sectional areas, decreasing the amount of molten metal to be re-melted.

The major factor limiting the gate velocity is the injection system of the die casting machines. The Die Line (equation (2.5)) is used to determine the pressure required by the die.
Equation (2.3) shows that if the gate velocity is doubled, four times more pressure is required from the injection system.

The fourth step relates to the establishment of the minimum and maximum gate depth. The minimum gate depth is calculated according to the density of the alloy, the gate velocity and a derived constant for the alloy in use. It is recommended that its value should not be smaller than 0.5 mm for machining considerations because the machining tolerances become a larger percent of gate depth if this value is exceeded [18].

The maximum gate thickness is constrained by mechanical factors such as the casting thickness. It is recommended that the gate depth must never exceed the casting thickness. Usually its maximum value is taken around 80% of the casting thickness. The gate thickness should also not unduly increase costs of trimming and finishing of the casting.

It is important to mention that if there is more than one gate, each gate has to be evaluated independently.

To determine the feasibility of the design to produce the desired casting, the die casting machine capabilities and the die requirements must be matched. This constitutes the fifth step in the methodology. It must be known or specified, the flow rate, the plunger velocity, the total apparent
gate area\textsuperscript{1}.

Matching the die requirements and machine capabilities can be achieved by two alternative approaches [8]. One of them decides the fill conditions in terms of the gate velocity and filling time. This approach is useful when a comparison among several machines is needed. It can also be used to decide the optimal plunger diameter and working pressure.

The second one, uses a given machine with a particular plunger diameter and operating pressure and afterwards the range of filling conditions are analyzed in order to determine their suitability to the production of the casting. This approach is beneficial if a limited number of machine types and plunger diameters are in use. It does not require changes in the operating pressure. Furthermore, the effect of varying the gate area can be examined. Also, the final decision on filling conditions can be deferred until all the possibilities are analyzed.

For both approaches the P–Q\textsuperscript{2} diagram is used in order to determine the feasibility of the design. The Machine and Die Lines are used to decide this issue. The intersection point between these two lines determine the

\textsuperscript{1}This is the area if the gate is positioned perpendicular to the flow
maximum flow rate that can be achieved with this die casting machine in order to produce the casting represented by the Die Line.

There exist several guidelines for obtaining better results in the production of castings. It considers the use of the smaller cross sectional area of the runner and gate to minimize the power requirements. A constant cross sectional area eases the die build although it will require a minor increase in power. A substantial reduction in the power required from the die casting machine can be achieved if the shot weight is decreased. It is also recommended the available machine power be used in order to reduce the the runner size.

The plunger diameter and the speed are considered next. The final selection of the plunger diameter is influenced by factors such as die flashing, wear and tear of the die, the amount of air entrapped in the molten metal, pressure feeding shrinkage and size constraints.

Die flashing relates to the temperature gradient required inside the die in order to allow the solidification of the molten metal and to the pressure applied on the molten metal inside the cavity. Temperatures gradient cause changes in shape of the die. The pressure is responsible for die die flashing. This is a stage where the internal forces surpass the clamping forces of the
die resulting in a little stretch of the machine's tie bar and a small separation between the die halves.

The pressure on the molten metal strongly depends on the plunger diameter; the larger the plunger diameter, the smaller the die flashing. Also, a reduction in the plunger velocity decreases the kinetic energy that has to be dissipated when the fast shot is finished and the die flashing. Furthermore, the impact peak pressure water hammer is reduced since a smaller hydraulic pressure is applied to the molten metal.

Although it is difficult to quantify die wear and tear, it is known to be related to the impact peak pressure. The peak pressure causes hydraulic leakages and deflections in the machine. The clamping system is also affected by the extra stretching in the tie bars. This behaviour in turn accelerates the die wear. Usually, increasing the plunger diameter reduces the tear and wear of the die.

On the other hand, the choice of a larger plunger diameter increases the wear in the shot sleeve. In a shot sleeve with a bigger diameter the thermal gradient increases since the molten metal occupies a small fraction of the shot sleeve volume and a less uniform temperature distribution exists. As a result, the bottom of the shot sleeve could have a high temperature than the
top. This in turn affects the bore through which the plunger moves when
the shot is made and causes wear of the plunger and the shot sleeve.

The shot sleeve life is also affected by washout, soldering and heat check-
ing [4]. Washout occurs due to the gradual loss of shot sleeve material directly
under the pouring hole. A higher production rate influences the available
time for the shot sleeve for distributing and losing heat. As a result of local
overheating the sleeve iron dissolves into the molten metal. It is also affected
by an excessive molten alloy superheat.

The development of metallurgical bonds between the sleeve and the cast-
ing alloy is called soldering. It usually occurs when a thin layer of the casting
alloy is permanently bonded to the shot sleeve. This causes scoring, stick-
ing and wear of the plunger tip. Soldering is usually associated with sleeve
overheating. It is affected by improper lubrication of the sleeve and plunger
tip and the existence of sludge or residual flux related product in the casting
alloy.

The amount of trapped air increases when a larger plunger diameter is in
use. Since the molten metal does not completely fill the shot sleeve. a bigger
shot sleeve means that a larger volume of air has to be exhausted through
the air exhausts. The extra air increases the possibilities of gas pockets and
porosity in the casting and demands a better control of the slow shot speed.

Die casting practice shows that usually an additional amount of molten metal is added to the cavity after the filling using the force of the plunger while the solidification shrinkage occurs. This extra pressure allows better castings when aluminum alloys, magnesium and brass are utilized. It is also effective for the production of thick castings. The relationship between the pressure on the plunger face is inversely proportional to the plunger diameter (equation (2.8)). Then if the plunger diameter increases, the pressure on the plunger face will decrease. Then, the effect of the plunger pressure on pressure feeding shrinkage diminishes.

The minimum and maximum diameter of the plunger have to be considered. The minimum value depends on the column strength of the shot rod and coupling. The maximum value is a function of the hole through the platen and the allowable minimum of the shot sleeve wall thickness. If no plunger diameter can be found within these boundaries, the machine should not be used.

The calculation of the final runner and gate dimension ends the design methodology. At this point, the actual gate area and the selection of the gate depth have to be made. These calculations depend on the gate type. The
final runner volume and the sprue/biscuit volume and the total shot volume are calculated.

In this step, the calculation of the slow shot velocity is also performed. It requires the percent of the initial filling of the shot sleeve and the total shot volume.

The present work does not cover all the steps of the above methodology. It mainly deals with several issues subject to improvement.

4.2 Proposed Approach

During the description of the methodology in the previous section several topics subject of improvement or inaccuracies have been highlighted. The proposed methodology improves the accuracy of the calculation in order to provide better tools for producing casting of better quality.

This methodology uses values accurately obtained from calculations based on a continuum mechanics formulation of the die casting physics. It is not intended to change the sequence in which the calculation are performed but how the values are obtained. Furthermore some additional factors such as the static pressure from the air exhaust lines and the optimization of the
gate area and plunger diameter are considered.

The freezing time can be accurately calculated based on a continuum mechanics formulation by a numerical analysis of the energy analysis of the casting cycle, casting to die (see Appendix B). It requires the casting geometry, cooling lines location and die closed and open cycle time. Once the freezing time is known, the maximum filling time has to be smaller than the freezing time in order to fill the die before the molten metal freezes.

The calculation of the feasibility involves the Die Line and Machine Line. The calculations related to the Die Line are performed as follows:

Given the runner and gate design, the viscous pressure drop in the runner and gate can be computed by a numerical analysis with the Navier-Stokes equation for the runner and gate. This provides the information to define the Die Line.

To improve the flexibility of the design and the possibility of further optimization, the vertical boundaries of the Operational Window, i.e., the maximum and minimum filling times should be far enough apart. This can be achieved by setting the maximum filling time as a large fraction of the freezing time.

Given the design of the air exhaust system, the filling time and the flow
rate, the mass flow rate of air through each air exhaust can be computed. Then, the static pressure from each air exhaust line and the minimum filling time are also computed.

These two pressures, due to the Die Line and the set of air exhaust lines can be added to produce the Die Design Line (DDL). This new line considers the effects of both the viscous pressure drop and static pressure from the air exhaust set. The DDL can be plotted on the P-Q^2 diagram.

On the P-Q^2 diagram the intersection of the Die Design Line with the Machine Performance Envelope or the Machine Line tangent to it, represents the minimum filling time for the given die casting machine and die design. Furthermore, the intersection point determines the plunger diameter. The gate velocity can be calculated since the gate area, the volume of metal through the gate and the filling time are known.

It is worth noting that the feasibility of the design checks that the machine has enough power to produce a given casting. On the P-Q^2 diagram that means that the pressure on the DDL for a given flow rate is smaller than the pressure on the Machine Line for the same flow rate.

The above described methodology can be summarized as follows [11]:
1. Given a casting, cooling lines, die open and die closed cycle time. Calculate the freezing time with the numerical analysis.

2. Given the freezing time, compute the filling time and the gate area within the boundaries of the Operational Window.

3. For a given air exhaust set and filling time compute the static pressure from the air exhaust lines.

4. Given the runners and gate compute the Die Line using the numerical solution of the viscous flow of the Navier-Stokes equations.

5. Given the runners and gate design and the filling time solve the energy equation during the filling stage.

6. Use the solution of the advective energy equation to estimate the temperature of metal while the die is being filled. Compute a new freezing time with the calculated temperature in step 1. If the freezing time has changed, return to step 2.

7. Compute the Design Die Line and the pressure required for this design $P_d$. 
8. Select a machine and obtain the Machine Performance Envelope. Determine the maximum pressure the machine can provide \( (P_m) \) for the given filling time.

9. If \( P_d > P_m \), then the machine does not have enough power to produce the given casting. Therefore either increase the gate area, increase the air exhaust set, increase the filling time or select a more powerful die casting machine.

10. If \( P_d < P_m \), the machine has enough power. If the optimum plunger diameter has not been used, optimize the plunger diameter. The gate area or the filling time can be decreased or optimized to strive for a higher quality of casting.
Chapter 5

Computational Implementation

This chapter introduces the computational implementation of the Parametric Design module and the static and evolving design scenarios. It also presents an example of how the evolving design scenario can be used to design castings successfully.

5.1 Software Design

In this work, the die casting process knowledge space is divided in two parts [10]. One part deals with the information related to the die. The other part, deals with the remaining information needed in the die casting design
process. This is schematically shown in Figure 5.1.

It can be mathematically expressed as:

\[ S_{dc} = S_{die} \cup -S_{die} \]  \hspace{1cm} (5.1)

where:

- \( S_{dc} \): knowledge of die casting process space;
- \( S_{die} \): knowledge of the die space;
- \(-S_{die}\): complement of the knowledge of die space.

The knowledge of the die space includes the die, the runners and gate the overflows and air exhaust lines geometries, the pressure drop in the runners and gate, the gate velocity and area, etc. Consequently, the complement set includes the complement information, i.e., the information not included in the die space. This space includes information related to the die casting machine such as the dry shot velocity, the diameter of the shot sleeve, the plunger velocity, the pressure available from the accumulator, the position of the pouring hole, etc.

The knowledge of the die casting process space \( S_{dc} \) can be considered the
Figure 5.1: Die Casting Process Knowledge and its decomposition.

Operational Window, since this is where the feasibility analysis is performed based on the information available from each knowledge space.

This formulation is a natural extension of the basic principles described in Chapter 2. There it was shown that the Machine Performance Envelope is independent of any plunger diameter and the die mounted on the die casting machine. There is a clear boundary between the knowledge space related to the die and its complement.

Figures 5.2 and 5.3 depict the individual components of both spaces. These components were used for the implementation of the classes.

Object Oriented Programming (OOP) techniques are used in the design and implementation of the parametric design code. Specifically, a top-down design [5] was utilized to address the design at progressively lower levels of
Figure 5.2: Die Knowledge Components.

details. The design also considers programming issues such as modularity, modifiability and fail-safe programming.

Figure 5.4 shows the relationship among the different modules. The division of the modules is based on the server-client criteria. For instance, the Shot Sleeve module has as clients, the slow and fast shot and the shot sleeve geometry. At the same time Shot Sleeve becomes a client of the Air Exhaust module since the information about the shot is needed to calculate the mass flow rate and the outlet static pressure in the air exhaust system.
Figure 5.3: Complement of the Die Knowledge Components.

Figure 5.4: Bipartite graph showing the relationship among different modules.
5.2 The Design Environment

A decision on the feasibility of the design requires the integration of the individual modules. This generates the need for a design environment that provides tools for analysing a given design.

There are two different design scenarios. One static which allows an evaluation of the given design and other evolving that allows a progressive design based on the tools available to the die caster designer.

The static scenario checks the feasibility of the design based on the information supplied by the user but the information does not suffer any modification. It considers the parameters unchanged and only provides the information about the feasibility to produce castings of good quality under the given conditions.

On the other hand, it is possible to furnish the user with tools that permit a progressive definition of the design and support the decision making process with accurate information. To reach such goals, the design approach has to be flexible enough to allow the user to perform the calculation or evaluation in a non-predefined and interactive form. The function of the design environment is to provide appropriate tools to help make proper decisions, but does not
ties the user to a rigid procedure. It leaves the organization of the calculation procedure to the user.

In a highly specialized working environment such as die casting design practice, the user knows how to make correct decision, given the results. Therefore, the emphasis here is to provide accurate tools to make the decision process easier and more knowledgeable. It does not intend to substitute for the enormous amount of experience and knowledge existing in the die casting industry. Although it could be also used for training purposes if desired. This is the fundamental assumption that underlies the design environment.

From the implementation point of view, the design environment can be provided to the user by the integration or embedding of C++ code with the Tcl language [16].

Tcl language is a command language used for developing interactive applications [17]. Also it allows the extension of a basic set of commands with application-specific commands. A fully programmable command language is obtained as a result of this combination. Tcl language has a simple and minimal syntax and it provides basic features such as variables, procedures, control structures, etc.

Figure 5.5 shows the general idea about the level of interaction among
the different components or commands. Each component is available in the Tcl shell and can be used by the designer during the design section.

Parametric design is used as a *glue* that could drive the design procedure if needed. It interacts with the other modules to obtain the information required to decide about the feasibility of the design.

### 5.3 The Evolving Scenario

This section outlines one possible approach in which the evolving design scenario could be used for designing a given casting. The procedure requires the casting, overflow and runner geometries.
1. Be sure that the data in the design file is correct. Check for the pressure of the machine, dry shot velocity and others machine related parameters. That is an important decision which could affect the complete design but it can be changed if required.¹

2. Set the number of air exhaust lines (AELs). This selection is based on facts like the volume of casting and shot sleeve. At last what the AELs do is allow the air contained in the die cavity be pushed out of it. Say 5 AELs. There can be any number for starting, (the better the experience, the closer to the final number this number will be).

3. Set the shot sleeve geometry. Die casting machines come with a set of shot sleeves, which can be changed to satisfy different requirements.²

4. The casting and overflow geometries and the volume are known at this stage. then

\[ V_T = 1.05 (V_c + V_o) \]

¹This step can be executed at the end, if all the parameters are set and try to find a machine that will accommodate them.

²It is possible to define a fictitious shot sleeve, in the sense that it does not belong to any machine and then try to find the machine nearest to this shot sleeve. It should be carefully done since it is possible to end up with a useless shot sleeve geometry.
where:

$V_T$: total volume of liquid metal;

$V_c$: volume of the casting;

$V_o$: volume of the overflows.

This 5% increment tries to consider the volume in the runners, gate(s) and biscuit.

5. Knowing this $V_T$ and that the shot sleeve is usually filled to about 50 to 60%. A tentative length of the shot sleeve ($L_s$) can be set as follows:

$$V_T = 0.25 \pi (D)^2 L_s f_p$$

where:

$f_p$: filling percent of the shot sleeve;

$D$: diameter of the shot sleeve;

$L_s$: length of the shot sleeve.

This length $L_s$ is the useful length, that is the length from the pouring hole to the end of the fast shot. The physical length of the shot sleeve
should be greater than this value, considering that the pouring hole should not be at the very beginning of the shot sleeve.

6. Set the fast and slow shot velocities.

The fast shot velocity is constrained by the dry shot velocity. It cannot be greater than the dry shot velocity. Also, the closer this value is to the dry shot velocity the smaller the pressure available from the machine. This comes from the fact that the Machine Line is a line with negative slope. The first guess for the fast shot velocity can be about 50 percent of the dry shot velocity. This first guess can be further refined if needed.

The slow shot velocity can be about the 30 to 40 percent of the dry shot velocity. If this velocity is fast enough, wave will develop during the slow shot and it will cause porous castings, due to the air in the shot sleeve trapped with the liquid metal.

7. Set the fast and slow shot distances. The fast shot distance can be set to about 40 to 50 percent of the shot sleeve distance. Then, calculate the time required for the fast shot ($t_{fast}$), assuming URM of the plunger.
8. Knowing fast shot distance and $L_s$ (step 5) the slow shot distance and the time required for the slow shot ($t_{slow}$) can be calculated assuming URM of the plunger.

9. Calculate the filling time as follows:

$$t_{fill} = t_{slow} + t_{fast}$$

where:

$t_{fill}$: filling time.

Alternatively, the filling time can be calculated, considering volume of liquid metal through the gate.

10. Calculate the static pressure on the air exhaust lines. Assume the pressure drop in the runners and gate between 1 and 5 percent of the static pressure from the air exhaust lines.

11. Calculate the cold shut freezing time from the total volume of molten metal and the flow rate as follows:

$$t_{freezing} = V_T/Q_{fast}$$

where:

$Q_{fast}$: fast shot flow rate.
12. Select a gate area within the boundaries of the Operational Window.

13. Run the first trial of Parametric Design. Check the results to see if there is enough power from die casting machine to produce the casting. If enough power is available go to the following step. Otherwise increase the number of air exhaust lines, keeping the other parameters constant. If no solution is found, either the shot sleeve geometry or the die casting machine has to be changed.

14. Run the thermal analysis code and obtain the casting's temperature distribution in the die. This provide useful information on where the cooling lines can be located (in the hottest places, subject to constraints).

The location of cooling lines should try to avoid large temperature differences near the die surface. Special attention should be paid to the maximum and minimum temperature on the die surface.³

15. Run the Energy solver and obtain the correct die closed freezing time for the given die closed time.

³Here practical problems arise on how to locate the cooling lines but there is not enough information available at the current time.
16. Run the filling solver, check if the air exhaust lines are located in the last places to be filled. If not, change the air exhaust lines to these positions considering that they cannot be located outside the parting plane.

17. Check in the Operational Window if the gate velocity is within the recommended boundaries and that the freezing time is greater than the filling time, otherwise the casting will freeze before it is filled.

18. Run Parametric Design again with the new values, and check for the feasibility of the design.
Conclusions

1. A more accurate formulation of the pressure required by the die which considers the influence of the static pressure from the air exhaust system has been proposed.

2. The results from the filling, thermal and advective energy analysis are included in the design procedure to permit a more knowledgeable decision making during the casting design process.

3. A software module for performing the required calculations of the Parametric Die Casting Design has been developed.
Bibliography


Appendix A

The Air Exhaust System

Air must be removed from the die cavity during the filling and before solidification of the molten metal in order to avoid casting defects. To vent air, air exhaust lines are provided between the mating die halves.

The injection of the molten metal into the die cavity pushes the air out through the air exhaust lines. The air is compressed in the die cavity. This increase in pressure must be overcome by the liquid metal during injection. Pressure effects can be minimized by filling more slowly.

Section 3.1 explained how the minimum filling time boundary of the Operational Window depends on the air exhaust system. To set the minimum filling time boundary, the maximum allowable mass flow rate of the air pushed
out from the cavity has to be calculated. Once the maximum mass flow rate of air is known, the static or stagnation pressure of air in the cavity as a function of time can be obtained.

The air velocity measured relative to the speed of sound is the Mach number. Surpassing a critical value of the pressure causes the development of a shock wave at the air exhaust outlet. This limits the maximum velocity of the air through the conduit. The mass flow rate could still increase with a higher pressure from the plunger since the density increases. A model has been proposed [13] in order to calculate the mass flow rate of air.

The model makes the following assumptions:

1. The air exhaust length and cross sectional area are constants

2. The duct width/height and length/height ratios are large and therefore the fluid flow variables are considered to vary only in the x direction.

3. The pressure, temperature and density are uniform within any cross section of the duct.

4. The air is modeled as a perfect gas.

5. The air pressure, temperature and density in the die is constant at any
time, i. e., there is no spatial gradient.

6. The air compression in the cavity is adiabatic and isentropic.

7. The air in the cavity does not exchange heat with the molten metal.

8. The friction factor is constant throughout the length of the air exhaust duct.

The calculation of the mass flow rate is performed for unchoked and choked conditions based on the model shown in Figure A.1.

The following information is needed:

Gas general properties:

- gas constant: \( R = \bar{R}/\bar{M} \)
- sound speed: \( c = \sqrt{kRT} \)
- Mach number: \( M = \frac{v}{c} \)
- density: \( \rho = \frac{P}{RT} \)
- mass flow rate: \( \dot{m} = \rho A c M \)

\( A \): cross sectional area of the duct.

Hydraulic diameter:

\( R_h = A/C = \text{cross sectional area} \div \text{perimeter} \)

\( D_h = 4 \times R_h \)
Air properties:

\[ k = 1.4 \]
\[ \bar{R} = 8314.4 \text{ J/kg-mole K} \]
\[ \bar{M} = 28.97 \text{ kg/kg-mole} \]
\[ R = 287 \text{ J/kg K} \]
\[ c_p = 1005 \text{ J/kg K} \]

Choked Conditions:

1. \( \bar{M}_2 = 1 \).

2. Calculate \( (fL/D_h) \). Then \( (fL/D_h)^* = (fL/D_h) \).

3. From the adiabatic frictional flow in a constant-area duct table \([21]\),
   obtain \( M_1 \), \( (P_1/P_a)^* \) and \( (T_1/T_a) \).

4. From the isentropic flow of a perfect gas table \([21]\) obtain \( (P_1/P_{01}) \) and
   \( (T_1/T_{01}) \).

5. Calculate \( P_{01} \) and \( T_{01} \) using \( P_a \) and \( T_a \).
   \[ (P_a/P_{01})^* = (P_1/P_{01})(P_1/P_a)^* \] and \( (T_a/T_{01})^* = (T_1/T_{01})(T_1/T_a)^* \)
6. Calculate $\rho_{01}$ and $\rho_1$ and the mass flow rate through the air exhaust from the perfect gas law.

Unchoked Conditions:

1. Assume $M_2 < 1$ and obtain $(fL/D_h)^*$. $(P_1/P_a)^*$ and $(T_1/T_a)^*$

2. The duct would be choked if a duct of fictitious length, $L_1$, is added to the actual length $L$ (Figure A.2). The critical (total) length $L_2$ is:
   
   $$(fL_2/D_h)^* = (fL/D_h)^* + (fL_1/D_h)^*$$

3. Knowing the critical length, obtain the Mach number at the conduit outlet $M_2$ and $(P_2/P_a)^*$ and $(T_2/T_a)^*$ from the adiabatic frictional flow in a constant-area duct table [21].

4. Obtain $P_a$ and $T_a$, since $P_2$ and $T_2$ are known and equal to the ambient conditions.

5. Knowing $P_a$, $T_a$, execute the procedure for choked conditions.

The star (*) denotes the critical conditions where the Mach number is equal to 1. The stagnation properties are represented by the (0) subindex. The (a) subindex stands for the atmospheric conditions.
Figure A.1: Flow of air through an air exhaust after Karni [13].

The mass flow rate through the air exhausts is related to the plunger area and velocity. The movement of the plunger increases the pressure in the die cavity while pushing out the air through the air exhausts.

The initial volume of air is considered as the volume of the die cavity plus the portion of the shot sleeve after the pouring hole. As the plunger moves, the air volume decreases by the amount of air displaced by the plunger. At the same time, the mass of air decreases as a result of the air being expelled through the air exhausts.

Once the inlet Mach number is obtained, the Mach number, static pressure and temperature at any section of the air exhaust duct can be computed.

In most cases the hydraulic radius of the air exhaust equals half of its
Figure A.2: Total and fictitious lengths for the unchoked conditions.

thickness. The Mach number becomes a function of the air exhaust line thickness and length. For a set of n air exhaust lines, the total area can be calculated as the sum of each individual area.

Air exhausts are located in the last places in the die to be filled with liquid metal. This would allow the complete expulsion of the trapped air from the die cavity during the filling for any plunger velocity. Unfortunately, the location of the air exhausts is constrained to lie on the die parting line only and the difficulty of separating the molten metal and the air through a solid front does not allow the complete expulsion of air in all cases.
Appendix B

Overview of Continuum Mechanics

Section 3.1 explained how the Operational Window is constrained by the minimum and maximum filling times.

The calculation of the maximum filling time involves the application of continuum mechanics. This appendix provides a brief overview of the description of the most important topics related to this issue. This explanation is not intended to be complete or extensive since the complexity of this topic is out of the scope of this work. It is included here for completeness purposes only. Interested readers are encouraged to consult [9] for detailed formula-
The maximum filling time depends directly on the freezing time since the die should be filled before the molten metal freezes. Cold shuts occur in castings when a flow channel freezes before the cavity is completely filled.

Incompressible viscous flow is described by the Navier-Stokes equation in the non-linear time dependent form

\[ \rho \cdot u + u \cdot \nabla - \mu \Delta u + \nabla p = f \]

\[ \text{div}(u) = 0 \]

where \( \rho \) is the density, \( u \) is the velocity, \( p \) is the pressure, \( \mu \) is the dynamic viscosity and \( f \) is the body forces. The density and viscosity vary in time and space. The above expression is only completely defined when the geometry of the domain and the initial and boundary conditions are defined.

The hyperbolic partial differential equation for modeling the free surface evolution through the advection of the density field for the fraction of liquid and the fraction of can be expressed as follows:

\[ \dot{\psi} + u \cdot \nabla \psi = 0 \]

where \( \psi \) is the fraction of liquid function (VOF), and \( u \) is the velocity.
The model for the thermal-transport problem with solidification kinetics is considered as follows:

\[ \dot{h} + \kappa \Delta T + c_p (u \cdot \nabla T) - L f_s = s \]

where \( h \) is the specific enthalpy, \( \kappa \) is thermal conductivity, \( T \) is the temperature, \( \Delta T \) is the Laplacian, \( c_p \) is the specific heat, \( u \) is the velocity, \( L \) is the latent heat of diffusion, \( f_s \) fraction of solid per unit volume and \( s \) is the entropy over unit mass. The specific heat, latent heat and thermal conductivity vary in space and time.

The fluid flow and solidification of the die cavity filling can be modeled through solving the incompressible Navier-Stokes equation, the hyperbolic free-surface equation and the energy equation with solidification kinetics.